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AND

DISCUSSIONS

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* Publication of closing discussion pending.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

A METHOD OF COMPUTING URBAN RUNOFF

BY W. I. HICKS,[1] ESQ.

SYNOPSIS

The purpose of this paper is to present the results of certain hydraulic investigations and rainfall-runoff gagings for improved urban drainage areas of different sizes and types of development, to analyze data from these results, to develop hydrographs of runoff for areas of various sizes, development, and times of concentration, to present methods of computing runoff for use in design offices, and to compare actual values of runoff with those computed by these methods. Much of the complete research, upon which this paper is based, has been placed on file for reference in Engineering Societies Library.[1a]

BASIS OF RUNOFF COMPUTATIONS

Rainfall-runoff relations in Los Angeles, Calif., are characterized by (*a*) a winter rainy season with consequent low evaporation and soil losses; and (*b*) wide variation in rainfall intensities on account of the mixed mountain, valley, and coastal plain terrain.

For urban areas, runoff is almost wholly that part of the rainfall which is not lost by infiltration into the soil or left in surface depressions and on vegetation surfaces to evaporate or sink into the ground.

The hydrograph of the net rainfall before it runs off the lot has the pattern of the rainfall intensities modified by the losses. Between this point and the point of concentration in the drain, the pattern is altered further by detention, which is the varying volume of water that must be built up to induce flow at the varying intensities of runoff. The general classification of detention is overland, gutter, and conduit.

This theory of runoff has been advanced by a number of engineers: The late C. E. Grunsky,[2] Past-President, Am. Soc. C. E., advocated the modifica-

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NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by September 1, 1943.

[1] Civ. Engr., Storm Drain Div., Bureau of Eng., Los Angeles, Calif.

[1a] 29 West 39th Street, New York, N. Y.

[2] "The Sewer System of San Francisco, and a Solution of the Storm-Water Flow Problem," by C. E. Grunsky, *Transactions*, Am. Soc. C. E., Vol. LXV (1909), p. 294.

tion of net rainfall by storage; Robert E. Horton,[3] M. Am. Soc. C. E., made a complete analysis of the theory; W. W. Horner, M. Am. Soc. C. E., and S. W. Jens, Assoc. M. Am. Soc. C. E., applied the theory[4] to the runoff from a city residential block and suggested its projection into the drainage network.

I. ANALYSIS OF RAINFALL RECORDS

Intensity, Frequency, and Variation with Locality.—The Los Angeles County Flood Control District, H. E. Hedger, M. Am. Soc. C. E., chief engineer, has analyzed rainfall records operated by the District and other agencies to derive intensity-duration curves (Fig. 1) and an isohyetal map of the county showing the variation in the intensity of the 50-yr 1-hr duration over the mixed mountain and valley terrain. The map in Fig. 2 is based on studies of the Flood Control District, the U. S. District Engineer, and the writer (for the coastal region).

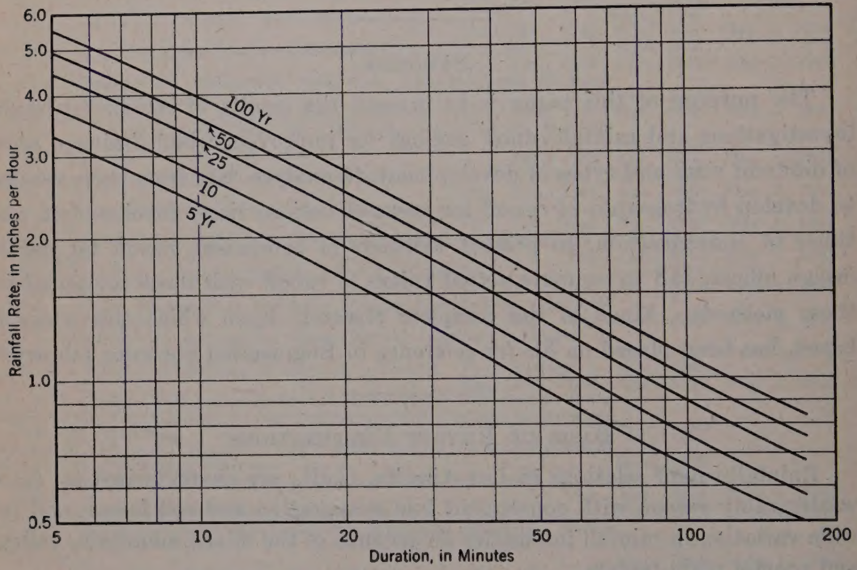

FIG. 1.—LOS ANGELES COUNTY FLOOD CONTROL RAINFALL

Storm Pattern.—The storm pattern was derived by plotting the major storms of the Los Angeles Station of the U. S. Weather Bureau as mass curves with the center of the most intense 5-min duration at a common point (90-min clock time). The result was a pattern storm between "medium" and "delayed" and is illustrated in Fig. 3.

The rainfall intensity-duration curve (Fig. 1) is based on the analysis of a great number of individual storms; there is nothing in the construction of the curves which implies that a storm having a 10-yr frequency for any stated

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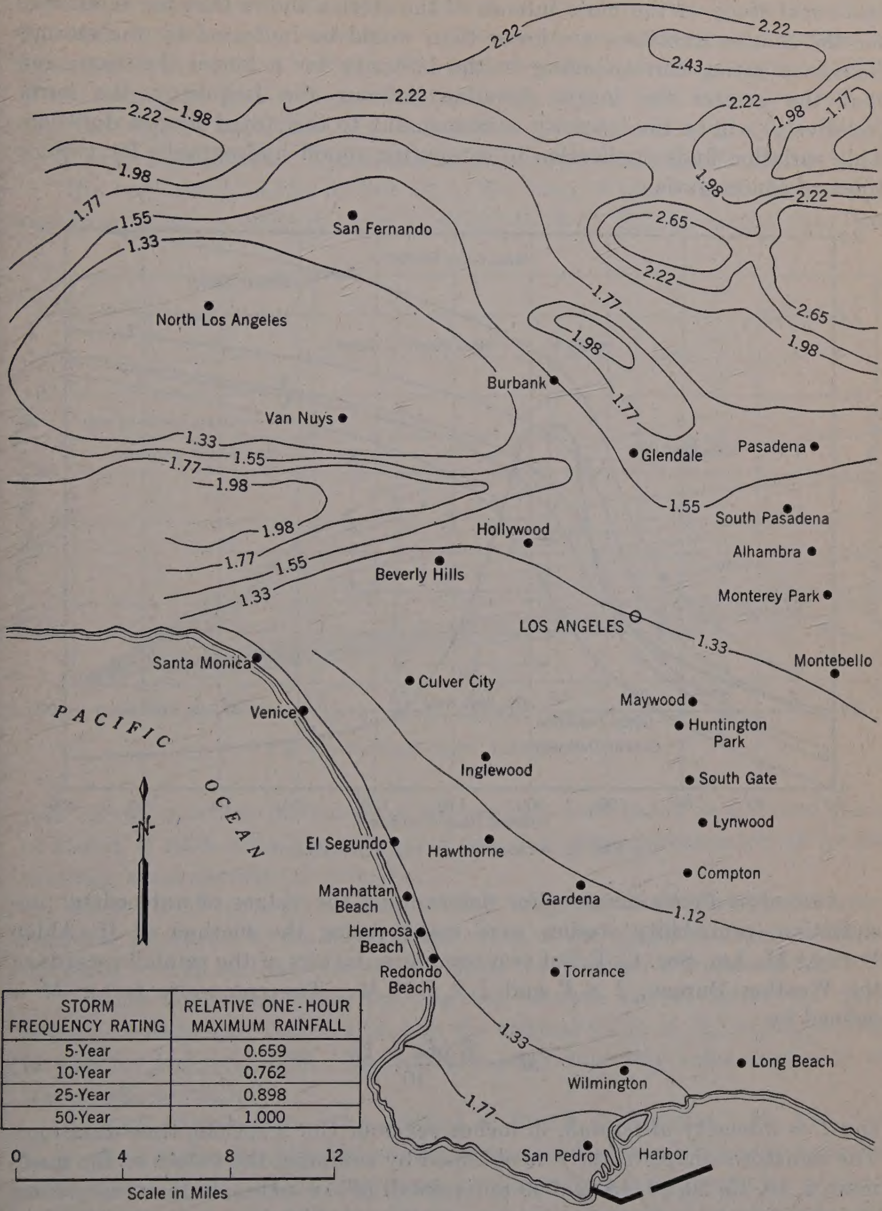
[3] "Surface Runoff Phenomena," by Robert E. Horton, *Publication No. 101*, Horton Hydrological Laboratory, Voorheesville, N. Y., 1935.

[4] "Surface Runoff Determination from Rainfall without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.



STORM FREQUENCY RATING	RELATIVE ONE-HOUR MAXIMUM RAINFALL
5-Year	0.659
10-Year	0.762
25-Year	0.898
50-Year	1.000

Fig. 2.—Isohyetal Map, Los Angeles Area

time duration will have the same frequency for any other time duration. A statistical study of the more intense of the storms shows that the intensities for the shorter durations are lower than would be indicated by the storm's frequency rating corresponding to the intensity for a longer duration; and that the greater the longer duration defining the frequency, the lower (relatively) will be the intensity corresponding to the stated shorter duration. This variation finds application in computing runoff hydrographs for various times of concentration.

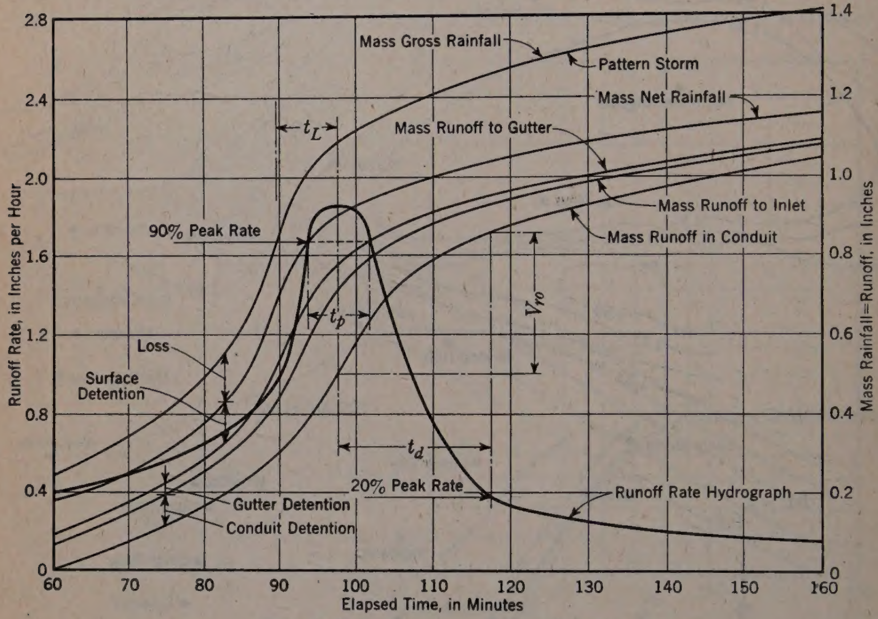


FIG. 3.—ELEMENTS OF THE HYDROGRAPH

Antecedent Precipitation.—For determining the values of antecedent precipitation, probability studies were made (using the method of H. Alden Foster,[5] M. Am. Soc. C. E.) of two composite factors of the rainfall records of the Weather Bureau, $I \times F$ and $I \times F \times M$. The composite factor M is defined by

$$M = \frac{E\,(d_{60} + 1)}{10} \quad \text{(1)}$$

and I = intensity of rainfall, in inches per hour (for a specific time duration). The rainstorm shape factor F is obtained by summing the values of the maximum 5, 10, 15, 20, 30, 45, and 60-min rainfall (d') in inches, in the order named and dividing by a corresponding summation of d''-values from the standard design rainfall curve (10-yr B curve, Fig. 1); or, $F_{15} = \dfrac{(d')_5 + (d')_{10} + (d')_{15}}{(d'')_5 + (d'')_{10} + (d'')_{15}}$

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[5] "Theoretical Frequency-Curves and Their Application to Engineering Problems," by H. Alden Foster, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

for $t_c = 15$, etc. The evaporation factor E is an arbitrary value based on the average evaporation for the 60-day period preceding the storm, derived from seasonal variation of evaporation. Antecedent precipitation, in inches, is expressed as $d_{60} + 1$, in inches. This is equivalent to the summation of 100% of the rainfall for 0 to 4 days antecedent, 50% of that for 5 to 9 days, 25% of that for 10 to 14 days, 15% of that for 15 to 30 days, and 10% of that for 31 to 60 days, antecedent. The residual soil moisture is allowed for by "+ 1" in.

The application of the divisor 10 to the term, $E\ (d_{60} + 1)$ to derive M, a moisture factor, is made in order to correlate M to Fig. 4,[6] rainfall-rate loss-rate curves, in which the values of soil-loss rate were based on the local storm of March 2, 1938, when the value of $E\ (d_{60} + 1)$ was approximately 10 for the drainage areas subject to analysis.

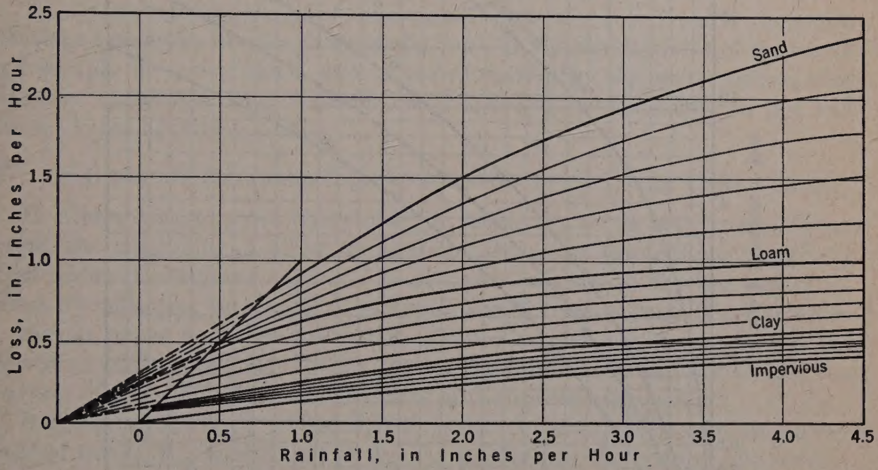

FIG. 4.—RAINFALL-RATE, LOSS-RATE CURVES

When $I\ F\ M$ for a given duration and frequency is divided by the corresponding product $I\ F$, the result is the value of the weighted moisture factor, M, equal to 0.39 on the isohyetal of 1.33 at the Weather Bureau station.

Because lawns and parks are irrigated between storms during the rainy season, an estimated 0.11 is added to the original value of M, making a total of 0.50 for isohyetal 1.33. The analyses show that this value applies to all frequencies of rainfall.

Rainfall Distribution.—The method of Frank A. Marston,[7] M. Am. Soc. C. E., for distributing rainfall intensities was applied to a large area centering in Pasadena, Calif., where records are available from a large number of automatic rain gages. These gages are located on isohyetals of varying intensity (see Fig. 2), and the records were corrected by the ratio of the value of the

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[6] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), Fig. 16, p. 1098.

[7] "The Distribution of Intense Rainfall and Some Other Factors in the Design of Storm-Water Drains," by Frank A. Marston, *ibid.*, Vol. LXXXVII (1924), p. 535.

50-yr isohyetal on which the selected central gage was located to the value of the 50-yr isohyetal on which each individual gage was located. The results of the study are shown in Fig. 5.

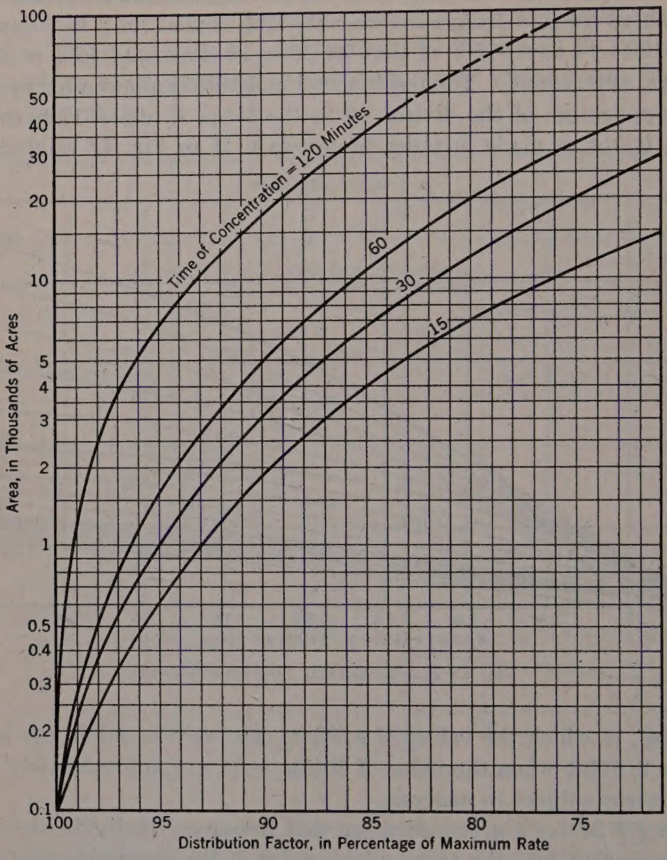

FIG. 5.—RAINFALL DISTRIBUTION CHART

II. SOIL INFILTRATION TESTS

A detailed report of soil infiltration tests on small soil plots with artificial precipitation was made in 1935 by L. W. Armstrong,[8,8a] M. Am. Soc. C. E. Another method was the analysis of rainfall-runoff records for losses from large partly pervious drainage areas. From these sources, the rainfall-rate loss-rate curves for a moisture factor, *M*, of 1 were derived for use in preparation of design data (see Fig. 4).[6] A value of 1 for *M* denotes that the soil is well saturated and that the lower and more stable values of infiltration capacity prevail.

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[8] Progress Report, Conference on Water Conservation, Los Angeles, California, by Committee on Conservation of Water, Irrig. Div., Am. Soc. C. E., December, 1935, p. 50.

[8a] Copy of paper will be furnished upon request, addressed directly to the writer.

In the paper by Messrs. Horner and Jens[4] and the discussion by the writer,[6] the question of variation of infiltration capacity with rainfall intensity was raised. Local experimentation shows predominant variation for bare soil as noted by the authors and a lesser amount of variation for sodded areas. The curves in Fig. 4,[6] used for design data, reflect the presence in urban areas of imperfectly graded lawns, bare soil, garden plots, and other departures from the plane surface on which any depth of detention spreads uniformly and utilizes the full infiltration capacity of the soil.

III. EXPERIMENTAL ANALYSIS OF OVERLAND FLOW AND GUTTER FLOW

For a thorough treatment of this subject the reader is referred to the aforementioned paper by Messrs. Horner and Jens, on the development of the runoff hydrograph for a city block, and to the discussion by the writer introducing details of experimentation made in Los Angeles on overland flow on turf and various types of paving surface.

IV. ANALYSIS OF RAINFALL-RUNOFF RECORDS FROM LOCAL URBAN AREAS

In attempting to solve analytically a problem with as many variables as runoff, the possibility of piling up accumulated errors through comparatively small errors in assigning values to these variables may be checked and minimized by collecting and analyzing rainfall-runoff records from drainage areas of various types and sizes. Further uses for rainfall-runoff records are the derivation of time and volume relations of the rainfall and the runoff, and the analyses of the shape and time elements of the runoff hydrograph.

Drainage Area Survey.—Drainage maps, improvement plans, and soil survey maps of the U. S. Department of Agriculture formed the basis of the survey. The drainage ridge was located by field surveys, and determinations were made of the percentages of pervious area A_p, impervious area A_i, and that portion of the impervious area directly tributary to the drain or gutter without flow over open soil A_d. Pertinent information is listed in Table 1. Runoff for Stations 10 and 11 was recorded with gutters calibrated from fire hydrant flow measured with a rated meter. All other runoff and rainfall were measured with automatic recorders using large scale charts.

TABLE 1.—CHARACTERISTICS OF LOCAL DRAINAGE AREAS

Station	Location of runoff station[a]	Area (acres)	Soil type	IMPROVEMENT (%)			Channel type
				A_i	A_d	A_p	
3	Eastern Ave. at Klamath St.	515.0	Clay and heavy loam	23.7	16.3	76.3	Gutter, pipe, reinforced concrete box
4	Broadway Channel[a]	1,549.0	Heavy loam	52.2	36.3[b]	47.8	Gutter, pipe, arch, lined channel
10	Spring St. at 5th St.	0.80	....	100	100	....	Gutter
11	5th St. at Spring St.	2.17	....	100	100	....	Gutter

[a] In Los Angeles, Calif., except Station 4 which is in Pasadena, Calif. [b] Estimated.

Time of Concentration.—This has been defined as the time for the storm water to flow from the ridge to the point of gaging, excluding finger-like pro-

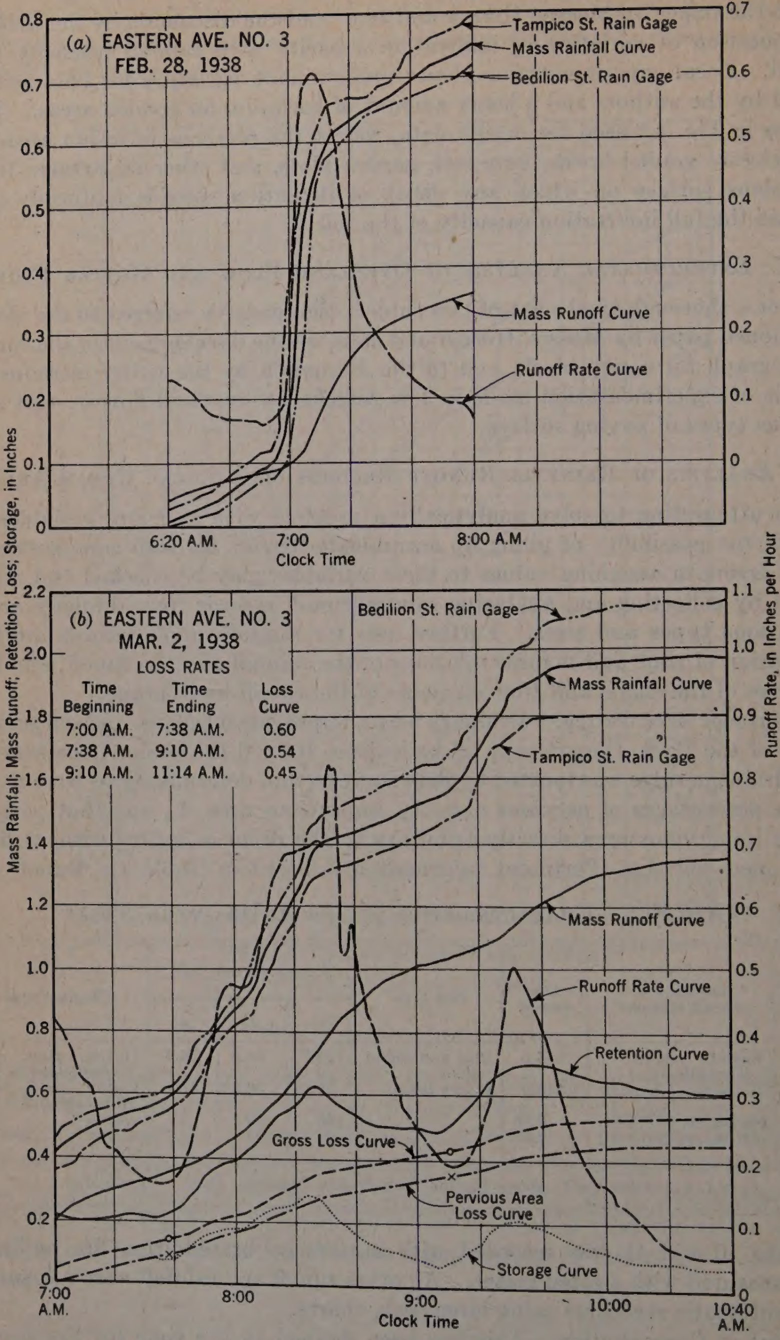

FIG. 6.—ANALYSIS OF STORMS RECORDED AT EASTERN AVENUE, STATION 3

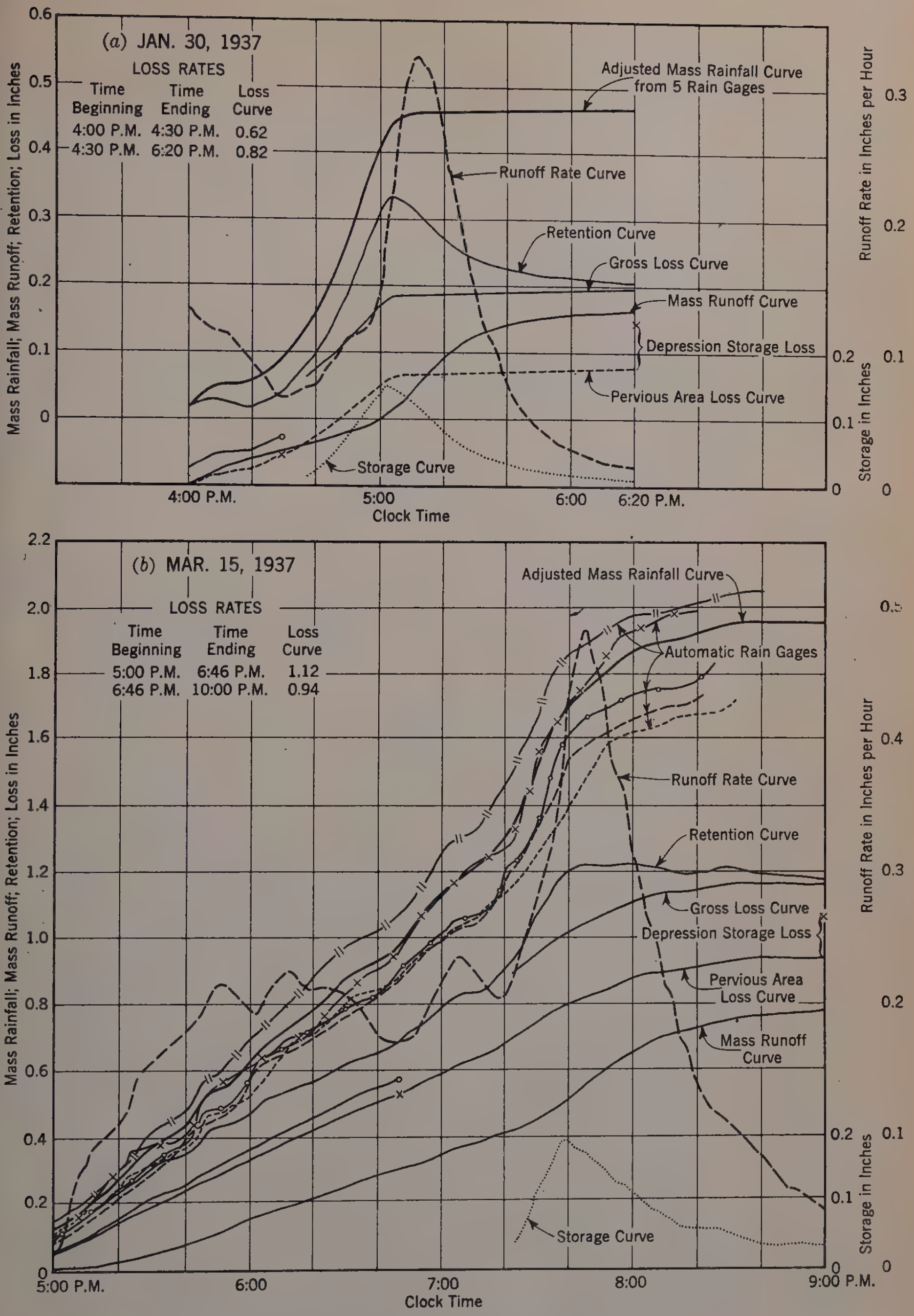

Fig. 7.—Analysis of Storms Recorded at Broadway, Station 4

jections of area. It is also the measure of an intercept on the mass rainfall curve which, with base flow, is effective in producing a peak of runoff.

In design work (for assumed conditions) the time of concentration consists of the inlet time for the initial block plus time of flow down the conduit (in minutes). For analysis of runoff records from partly pervious areas, the inlet time of the initial block is affected by the weights of the volumes of overland flow delivered from the lawns (slow flow) and from the impervious surfaces (fast flow). Account must be taken also of the fact that in normal design tabling, the time of flow in a closed conduit is computed for full-wetted perimeter (velocity = 86% of that for free flow); that the ultimate drainage system contemplates extension of the conduit to the initial block; and that flow in a well-designed drain is at efficient depth (free flow) except at points of hydraulic disturbance. Therefore, for analysis of records, the free flow velocities in conduits and gutters are multiplied by 0.86 in order to keep the data on the same basis as in design.

Time for overland flow is computed by the equation

$$t_c = \frac{60\,\delta_a}{\sigma} \dots\dots\dots\dots (2)$$

in which δ_a (average depth of flow) is computed for the various types of surfaces by formulas presented elsewhere;[9] and σ = rate of supply of excess rainfall, in inches per hour. Eq. 2 represents the horizontal (time) distance between the mass curves of rainfall supply and runoff. Gutter flow time is computed by[10]

$$t = \frac{0.0443\, l^{0.732}}{(S_o)^{0.39}\, \sigma^{0.252}} \dots\dots\dots\dots (3)$$

in which l = length of overland flow, in feet.

For partly pervious areas, such as Station 3 (Table 1), a series of time of concentration curves is needed, varying from a dry soil condition (about 18% runoff) to a saturated condition (about 76% runoff). Data for plotting these curves are presented in Table 2.

TABLE 2.—DATA FOR PLOTTING TIME-OF-CONCENTRATION CURVES

q_1	TIME, t_c, PERCENTAGE OF RUNOFF	
	76%	18%
0.15	59.7	32.3
0.20	54.8	30.9
0.40	43.2	27.0
0.60	36.5	24.3
0.80	32.3	22.7
1.00	28.7	21.7

Plotting.—Using a part of the storm of March 2, 1938, on gaging Station 3 (Fig. 6(*b*)) as an illustration, the mass rainfall curve (in inches), the runoff rate curve (in inches per hour), the mass runoff curve (in inches), and the retention curve or the difference between the mass rainfall curve and the mass runoff curve (in inches) need no further explanation for their method of plotting. The gross loss curve and the storage curve, both expressed in inches, are more involved and require more detailed explanation.

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[9] *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1100, Eqs. 17, 18, and 19.

[10] *Ibid.*, p. 1102, Eq. 20*b*.

First, select clock-time points on the record where the runoff rate is low and the rainfall rate has been low and fairly uniform for the preceding 20 min. With curves for the drainage area of depths of surface detention for low rates of rainfall and of depths of gutter and conduit storage for low rates of runoff, values of composite storage were plotted down from the retention curve; these points lie on the gross loss line and are control points.

Second, the value of impervious surface loss (experimentally determined as approximately 10% of the rain falling on the impervious surface) is computed for the reach between control points ($0.10\ A_i\ d_f = 0.0237\ d_f$ for Station 3, d_f being the volume of rainfall, in inches) and is plotted down from the control point at the right of the reach.

Third, the pervious surface loss is a function of the rain falling directly on the pervious surface plus the rain falling on the portion of the impervious surface discharging runoff on to the pervious surface; 10% of this latter rainfall is already lost on the impervious surface, leaving 90%, which will be subject to further loss on the pervious surface on to which it flows. The rainfall rate is adjusted therefore in the ratio: $\frac{A_p + 0.9\ (A_i - A_d)}{A_p}$, or 1.09, for Station 3.

Fourth, the reaches between control points are analyzed by the following check list:

(*a*) Time to time (for an intercept on the mass rainfall curve having an approximately uniform rate);
(*b*) Rainfall rate;
(*c*) Amended rainfall rate ($1.09\ I$);
(*d*) Infiltration rate (from Fig. 4[6]), for $1.09\ I$;
(*e*) Infiltration rate $\times A_p$ (76.3%);
(*f*) Impervious loss rate ($0.10\ A_i\ I$ or $0.0237\ I$); and
(*g*) Summation of items (*e*) and (*f*) (gross loss rates).

If the sum of the products of the rates in item (*g*) and the time intercepts in item (*a*) (in hours) does not equal the gross loss for the reach, recompute the tabulation with another infiltration curve from Fig. 4.[6] When a balance is secured, plot the pervious and gross loss curves. (The loss curves noted in Figs. 6 and 7 are identified by the loss intercept on a 3-in. rainfall rate in Fig. 4.[6])

Fifth, it is found frequently that, when a reach between control points contains high rates of rainfall and runoff, the infiltration curve required for balancing is greatly in excess of those used on both sides of the reach. Actually, the curve will lie between those for the reaches immediately before and after; the excess loss is depression storage. In this case, the tabulation is made for an intermediate infiltration curve; the products of items (*a*) and (*g*) are summed and deducted from the total divergence between control points; the result is a measure of depression storage loss. (For illustration, see Fig. 7 for Station 4.)

The storage curve, embracing surface, gutter, and conduit storage, is plotted as the vertical divergence between the retention curve and the gross loss curve. Analyses of other rainfall-runoff records are filed in the Engineering Societies Library.[1a]

Derivation of Hydraulic Phenomena.—A chart, showing a typical runoff hydrograph, its relation to the mass rainfall curve, and various factors to be analyzed in this paper, is delineated in Fig. 3. On the recession side of the hydrograph (see Figs. 6 and 7), when the depth of composite storage (D), in inches, is plotted against the coincident rate of runoff (q), in inches per hour, the resulting curve on rectangular coordinate paper is a straight line below 90% peak runoff rate. When the slope of this line $\left(\frac{\Delta D}{\Delta q}\right)$ is plotted against time of concentration, the result is also a straight line. Sample curves for Stations 3, 4, 10, and 11 are shown in Fig. 8. Fig. 9(a) shows the relation of the slopes of the storage-runoff rate curves and times of concentration as derived from analyses of records of Stations 3, 4, 10, and 11. Data are recorded in Table 3. In Col. 4, Table 3, percentage of runoff = ratio of the volume of runoff to volume of rainfall.

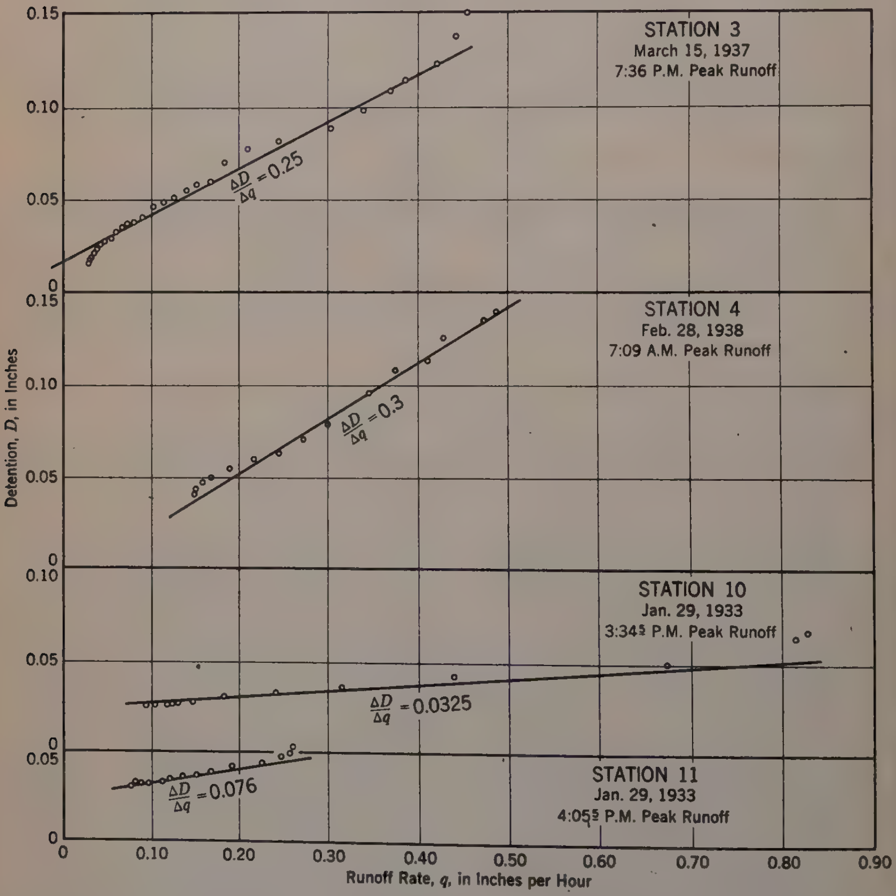


FIG. 8.—DEPTH OF STORAGE WITH COINCIDENT RUNOFF RATES, RECESSION SIDE OF HYDROGRAPH

In the analysis of storm records, it is necessary to reproduce the recession curve by computation as it would occur if the rainfall following 90% peak of runoff were in the adopted standard pattern (see Fig. 10 for Station 4, February 28, 1938). The net rates of rainfall adjusted in ratio to the standard pattern rates are 0.110 and 0.036; the value of storage at 90% peak is 0.125 in.; the time of concentration is 44.3 min. Using a value of $\frac{\Delta D}{\Delta q}$ (Fig. 9(*a*)) of 0.311 for 44.3 min, the theoretical recession curve is plotted to a point where the runoff rate is less than 20% peak:

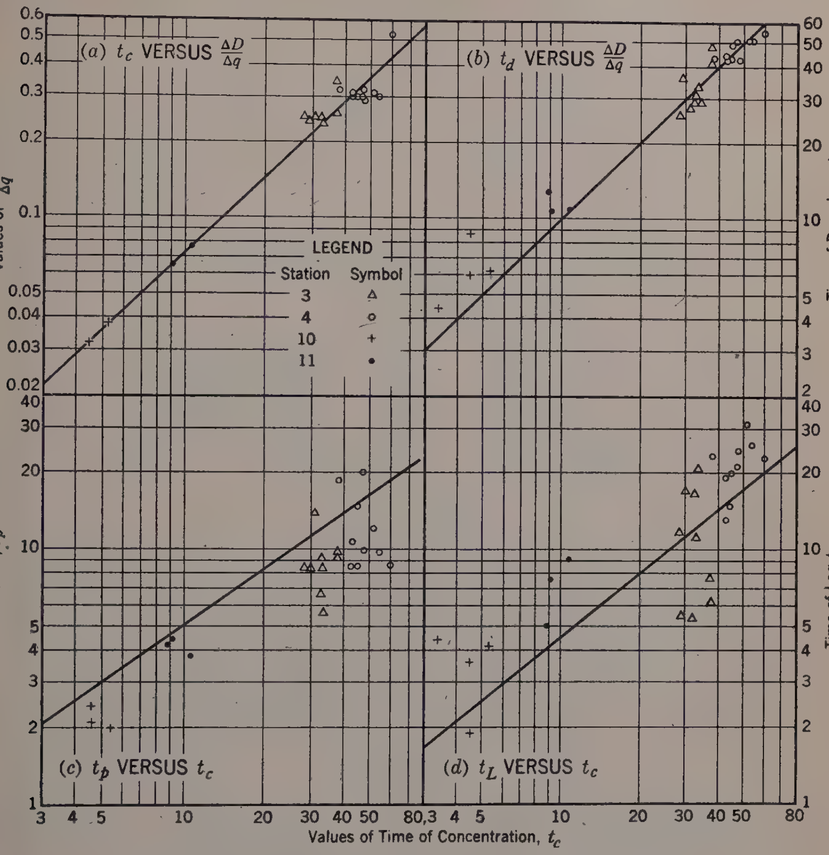


Fig. 9.—Time Elements of the Hydrograph from Records

1. From 7:13 to 7:14 a.m., plot the 90% peak runoff rate as a mass runoff curve, distant below the mass curve of net rainfall 0.125 in. (the value of storage), measured at 7:13½.
2. From 7:14 to 7:16, prolong the mass runoff curve with a rate (0.4224 in. per hr) so selected that the value of storage measured at 7:15, the center of the reach, lies on the curve of $\frac{\Delta D}{\Delta q}$ (0.117 in.).
3. Continue with 2-min reaches.

From these curves, the time of drawdown (t_d), or the elapsed time (in minutes) between 100% and 20% peak runoff rate, is measured (see Table 3 and Fig. 9(*b*)).

Another derived phenomenon of the runoff hydrograph is disclosed by plotting the recession curve on semilogarithmic paper. The recession curve after 90% peak tends to be a straight line. In Fig. 11, recession curves have been plotted, both revised as indicated in the preceding paragraph and without revision. It will be noted that, for the records having small times of concentration, the curve tends to draw to the right, away from a straight line, at a point well above the 20% peak runoff rate; for the long times of concentration, the curve remains a straight line down to a low percentage of peak rate. An explanation is that, as the time of concentration increases, the greater depth

TABLE 3.—ELEMENTS OF THE HYDROGRAPH DERIVED FROM RECORDS

Date (1)	Hour (2)	Record peak runoff rate, in in. per hr (q_1) (3)	Percentage of runoff (4)	Time of concentration (min) (t_c) (5)	$\frac{\Delta D}{\Delta q}$ (6)	Time of drawdown (min) (t_d) (7)	Time of peak (corrected) (min) (8)	Time of lag (min) (t_L) (9)
				(*a*) STATION 3				
1/ 5/35	3:44 a.m.	0.430	24.6	28.3	0.250	25.4	8.5	11.7
12/30/36	9:04 p.m.	0.320	34.2	33.4	0.240	29.2	5.6	20.6
3/15/37	7:36 p.m.	0.456	44.0	32.7	0.250	31.8	9.1	17.0
2/11/38	12:27 p.m.	0.315	30.4	32.2	....	29.3	6.7	5.4
2/28/38	7:08 a.m.	0.610	42.5	29.4	0.240	37.2	8.6	5.3
2/28/38	11:30 p.m.	0.560	56.0	33.3	0.240	33.4	8.4	11.4
3/ 2/38	8:32 a.m.	0.830	70.0	30.8	0.250	28.6	14.0	17.5
3/ 2/38	9:30 a.m.	0.505	69.0	37.5	0.260	49.2	9.8	6.2
3/ 2/38	3:54 p.m.	0.530	73.0	37.1	0.350	41.6	9.1	7.8
				(*b*) STATION 4				
10/18/36	4:21 p.m.	0.407	31.8	43.2	0.300	44.6	8.5	13.0
12/28/36	12:29 a.m.	0.156	51.1	60.0	0.530	55.0	8.5	23.6
1/ 5/37	7:34 p.m.	0.224	46.7	54.2	0.300	50.4	9.6	26.4
1/30/37	5:12 p.m.	0.325	41.4	48.0	0.290	43.4	9.9	24.5
3/15/37	7:45 p.m.	0.480	40.0	43.0	0.310	43.4	10.6	19.2
2/28/38	7:09 a.m.	0.497	44.0	44.3	0.300	43.4	8.5	14.8
2/28/38	11:45 p.m.	0.376	44.3	47.6	0.320	50.8	20.0	21.0
3/ 2/38	8:38 a.m.	0.880	58.5	38.8	0.320	43.4	18.6	23.4
3/ 2/38	9:41 a.m.	0.690	70.0	45.3	0.300	49.8	14.7	20.1
3/ 2/38	4:00 p.m.	0.557	80.0	52.0	0.310	50.8	12.0	26.6
				(*c*) STATION 10				
1/16/33	12:08 a.m.	1.570	90.0	3.4	....	4.4	1.3	4.4
1/16/33	3:36 p.m.	0.815	82.0	4.5	....	8.7	2.4	1.9
1/29/33	3:345 p.m.	0.828	78.5	4.5	0.0325	6.0	2.1	3.6
1/29/33	4:01 p.m.	0.560	69.0	5.3	0.0375	6.2	2.0	4.2
				(*d*) STATION 11				
1/16/33	3:40 p.m.	0.472	69.0	8.9	....	12.9	4.2	5.0
1/29/33	3:38 p.m.	0.432	66.3	9.1	0.064	10.8	4.4	7.6
1/29/33	4:055 p.m.	0.260	56.0	10.7	0.076	10.9	3.8	9.1

of water stored in transit governs the pattern of runoff for a greater length of time.

An equation for peak runoff rate has been derived from a study of the elements of the hydrograph (see Fig. 3):

$$q_c = \frac{K\,d_o}{t_d} \quad \text{. . . . .} (4)$$

in which q_c = the computed peak runoff rate, in inches per hour, K varies from 96 for a 5-min time of concentration to 108 for 60 min, and d_o = volume of runoff on the recession side of the hydrograph between 100% and 20% peak runoff rate, in inches of depth on the drainage area.

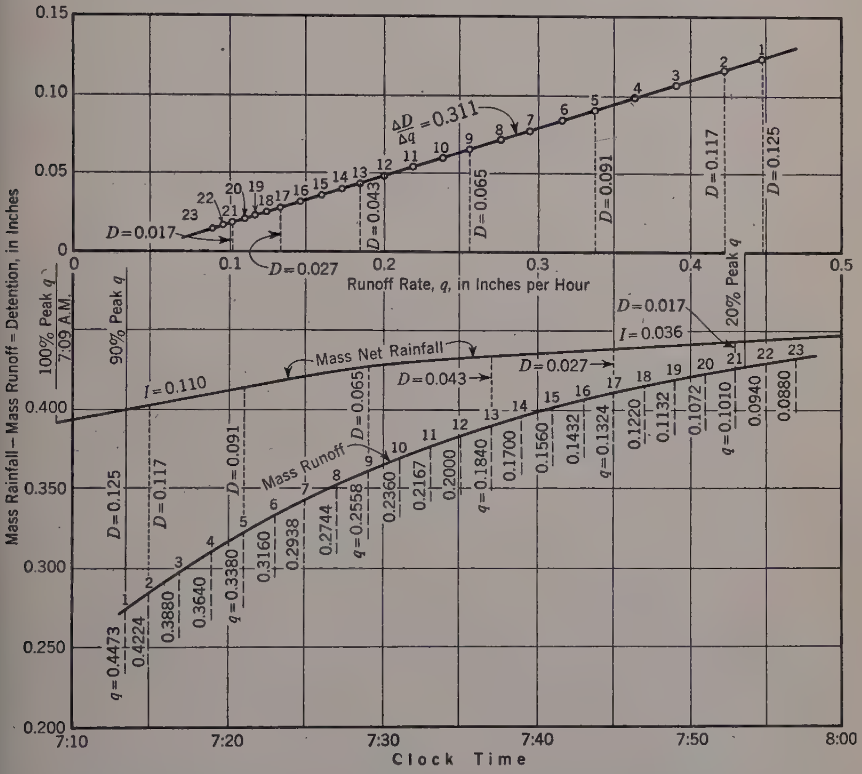


Fig. 10.—Computation of Theoretical Recession Curve (Station 4, February 28, 1938)

The foregoing hydraulic phenomena are characteristic of urban runoff but are not found in certain types of mountain runoff.

Derivation of Time Factors.—The meaning of the time factors is shown graphically in Fig. 3. Values derived from records are listed in Table 3. Comparison of their values with corresponding times of concentration is shown graphically in Fig. 9. The curves used hereinafter in the synthesis of the standard hydrograph are delineated as solid lines.

The lag time (t_L) can be defined as the time distance (in minutes) between the centroid of the intensities of the effective rainfall curve, for the intercept thereon of the time of concentration, and the time of peak runoff rate. It is dependent on the shape and distribution of impervious development of the

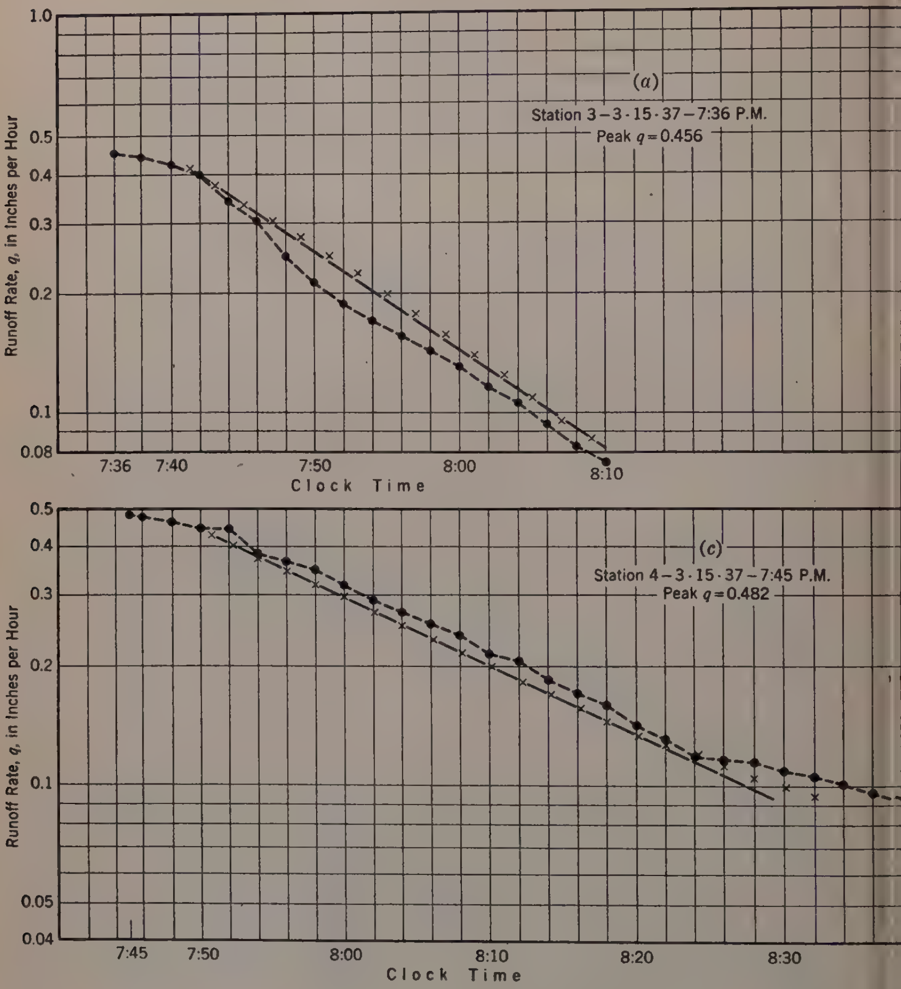
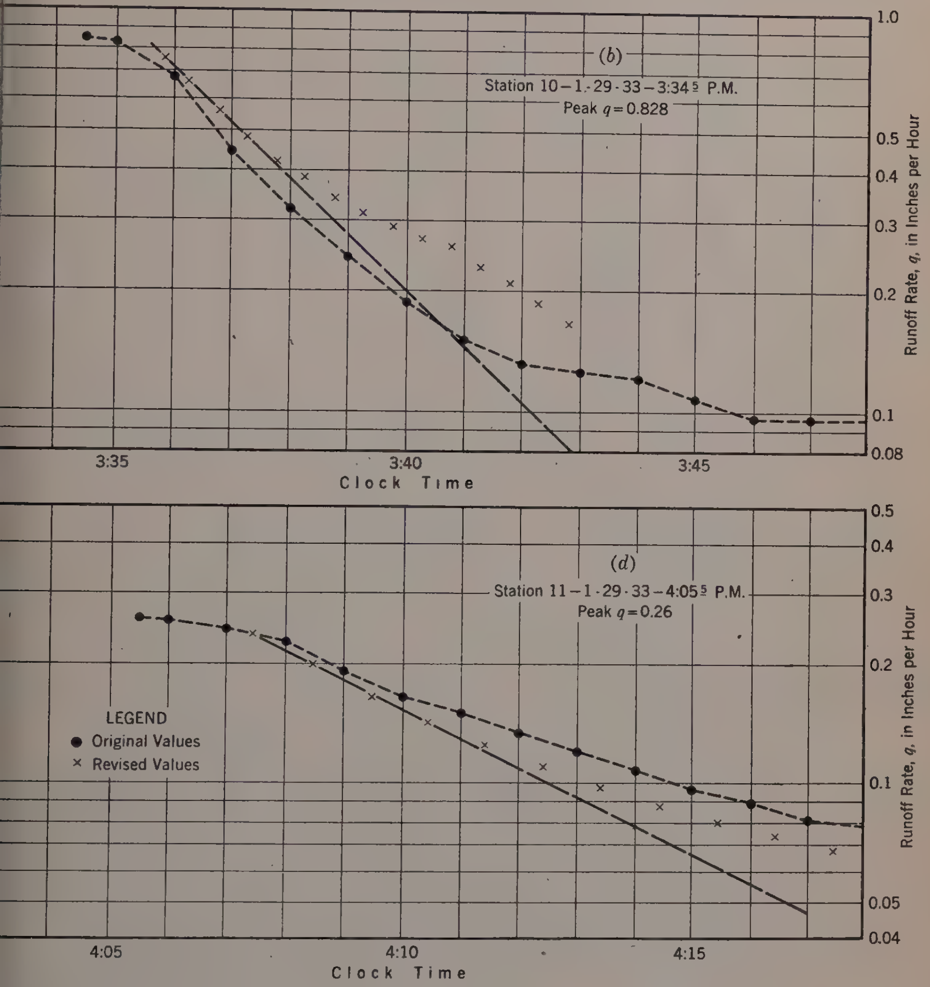


Fig. 11.—Ty

drainage area and no curve of fixed values can be derived from the data (see Fig. 9(*d*)). The time of drawdown (t_d) is shown by Fig. 9(*b*) to be approximately equal to the time of concentration for the adopted rainstorm pattern. The time in minutes from 90% to 90% peak runoff rate (t_p) is weighted by comparison with the time from 80% to 80% peak (see Fig. 9(*c*)).

V. RUNOFF FREQUENCY

The frequency of runoff is dependent in part on the amount of open soil in the drainage area and the wetness of that soil. This wetness is caused by precipitation antecedent to the high-intensity storm which produces the peak runoff. Infiltration tests and analyses of rainfall-runoff records show that runoff increases as the saturation of loams and clays progresses; this saturation is measured by the moisture factor *M*.



CESSION CURVES

Although sands, free of colloidal swelling, respond to an antecedent precipitation of much shorter duration, their infiltration capacity is so great that the use of the same moisture factor as for clays and loams causes no material error.

Analysis has been made of the factors in the equation

$$q_c = K_1 \, (I \, F \, M^{A_p})^x \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

for the 515 acres of predominantly pervious area of Station 3 (see Table 4).

TABLE 4.—DATA FOR EQ. 5 AT STATION 3

Date	Hour	Record peak runoff rate, in in. per hr (q_1)	q_1 corrected for prior storm (q_3)	Percentage of runoff	Time of concentration, in min (t_c)	Rainfall rate, in in. per hr (I)	Rainstorm shape factor (F)	Moisture factor (M)	IFM^{A_p}	Item No.
1/ 5/35	3:44 a.m.	0.430	0.413	24.6	28.3	1.19	0.805	0.243	0.33	1
12/30/36	9:04 p.m.	0.320	0.300	34.2	33.4	0.68	0.425	0.441	0.16	2
3/15/37	7:36 p.m.	0.456	0.422	44.0	32.7	0.88	0.484	0.372	0.20	3
2/11/38	12:27 p.m.	0.315	0.275	30.4	32.2	0.59	0.451	0.393	0.13	4
2/28/38	7:08 a.m.	0.610	0.570	42.5	29.4	1.09	0.790	0.377	0.41	5
2/28/38	11:30 p.m.	0.560	0.510	56.0	33.3	0.77	0.497	0.575	0.25	6
3/ 2/38	8:32 a.m.	0.830	0.730	70.0	30.8	1.06	0.600	0.838	0.55	7
3/ 2/38	9:30 a.m.	0.505	0.455	69.0	37.5	0.64	0.416	0.902	0.25	8
3/ 2/38	3:54 p.m.	0.530	0.460	73.0	37.1	0.55	0.430	1.105	0.26	9
12/18/38	7:43 p.m.	0.192	0.182	....	32.4	0.61	0.400	0.442	0.13	10
1/ 5/39	12:12 p.m.	0.229	0.219	....	30.0	0.73	0.403	0.215	0.09	11
1/ 7/40	7:53 p.m.	0.227	0.201	....	30.2	0.63	0.440	0.231	0.09	12
2/29/40	3:15 a.m.	0.260	0.250	....	31.0	0.64	0.405	0.305	0.10	13
3/31/40	4:47 p.m.	0.147	0.147	....	33.7	0.50	0.353	0.228	0.06	14

If $I \, F$ is considered constant, q_c varies as M (or M^{A_p}) to the exponent x. For Station 3, $K_1 = 1.00$ and $x = 0.55$. Based on studies of this and similar curves and on results of the trial-and-error development of the method of computation of runoff (see Fig. 12), runoff from open soil has been assumed to vary as $M^{0.59}$ for the range of rainfall intensities used in urban design.

VI. SYNTHESIS OF THE HYDROGRAPH FROM DERIVED DATA

Preliminaries to computing runoff hydrographs for use in design are assumptions for rainfall intensities (Figs. 1 and 2), antecedent precipitation (see Section I and Table 5), and classification of drainage areas as to impervious development. This classification is given in Table 6, which is based on surveys

TABLE 5.—MEAN VALUES OF THE MOISTURE FACTOR M

Isohyetal	1.12	1.33	1.55	1.77	1.98	2.21	2.65
M	0.450	0.500	0.555	0.612	0.665	0.730	0.835
$M^{0.59}$	0.640	0.675	0.715	0.755	0.795	0.840	0.910

TABLE 6.—CLASSIFICATION OF URBAN DRAINAGE AREAS

Type of development	SURFACE CLASSIFICATION			Factor of equivalent pervious area rainfall
	A_d	A_i	A_p	$\frac{A_p + 0.9\,(A_i - A_d)}{A_p}$
Park	15	15	85	1.00
Level residence	40	60	40	1.45
Hillside residence	50	50	50	1.00
Bungalow court	60	80	20	1.90
Apartment house	70	85	15	1.90
Impervious; business and manufacturing	100	100	0	....

and zoning ordinances. For brevity, the percentage A_d and the soil type (1 for clay, 2 for loam, and 3 for sand) are used to designate an area (15 − 1 denotes "park, with clay soil").

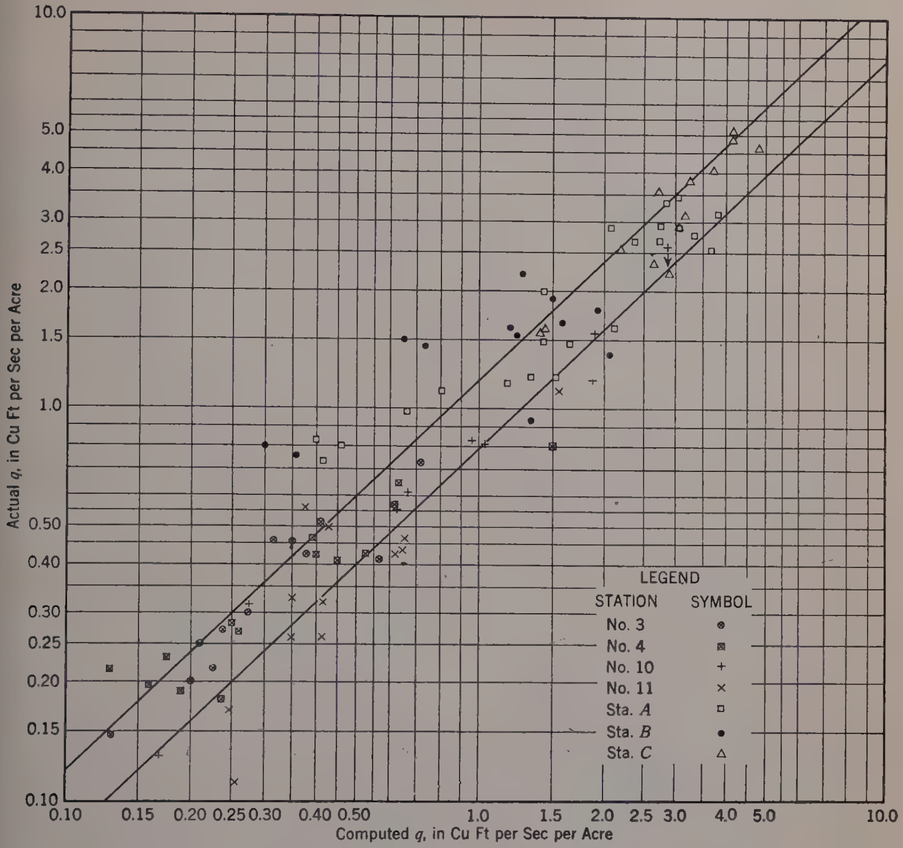


FIG. 12.—COMPUTED AND ACTUAL RUNOFF RATES—URBAN AREAS

The method of development of the runoff hydrograph follows (see Fig. 3):

1. Plot the gross mass rainfall curve (10-yr B curve, Fig. 1);
2. Deduct losses to form the net rainfall curve—
    (*a*) Impervious area losses;
    (*b*) Pervious area losses—depression storage and infiltration;
3. Deduct the varying depth of overland flow detention to form the mass runoff-to-gutter hydrograph;
4. Deduct the varying depth of gutter storage to form the mass runoff-to-inlet hydrograph; and
5. Deduct the varying depth of conduit storage to form the mass runoff-in-conduit hydrograph.

In the paper by Messrs. Horner and Jens,[4] the method has been developed down to, and including, the runoff-to-inlet hydrograph.

The writer has derived runoff-to-inlet hydrographs for the area classifications of Table 6 and for various types of soil. As the shapes of the resulting hydrographs were similar, they were plotted for the more intense portion of the storm on a common clock-time scale with percentages of accumulated depth of runoff as ordinates. The curves nested with slight variations; therefore the runoff-to-inlet hydrograph of the 10-yr rainfall curve on the 1.33 iso-

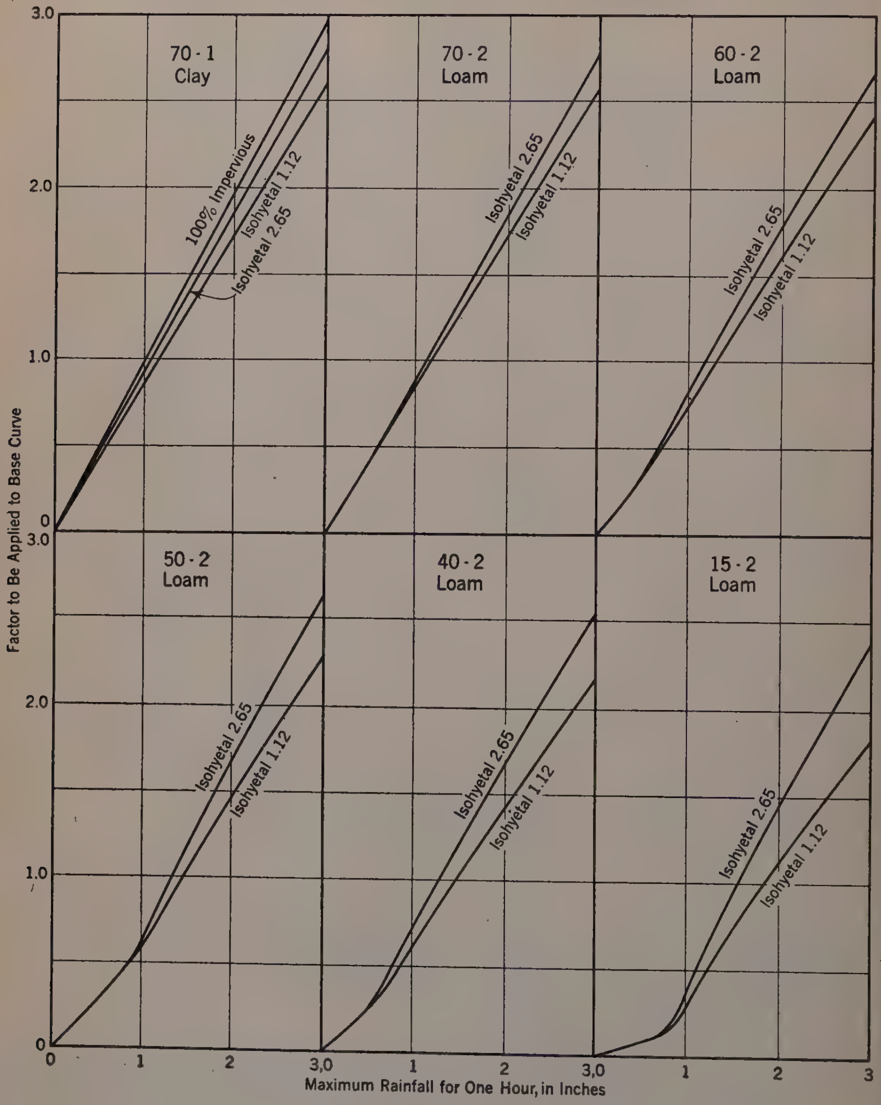


FIG. 13.—RUNOFF FACTORS

hyetal (Fig. 2) for 100% impervious development was selected as a base for the computation of hydrographs of runoff in the conduit, from which were derived the basic runoff rates for times of concentration from 5 min to 120 min (Table 7),

TABLE 7.—BASIC RUNOFF RATES AND REDUCTION FACTORS FOR CONDUIT DETENTION

(Col. A—Peak Runoff Rates Base Curve (Cubic Feet per Second per Acre); and Col. B—Peak Reduction Factor, Conduit Detention (Percentage per Minute of Flow))

t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)	t_c (min) (1)	A (2)	B (3)
5	3.40	12.00	14	2.17	4.10	26	1.65	2.04	42	1.34	1.18	58	1.17	0.80
6	3.13	10.00	15	2.10	3.80	28	1.60	1.88	44	1.31	1.12	60	1.15	0.77
7	2.93	8.60	16	2.05	3.50	30	1.55	1.75	46	1.29	1.05	70	1.07	0.64
8	2.76	7.50	17	1.99	3.30	32	1.51	1.62	48	1.27	1.00	80	1.01	0.55
9	2.63	6.60	18	1.94	3.10	34	1.47	1.51	50	1.24	0.96	90	0.96	0.47
10	2.50	5.90	19	1.90	2.90	36	1.43	1.42	52	1.22	0.92	100	0.92	0.42
11	2.41	5.40	20	1.85	2.75	38	1.40	1.32	54	1.20	0.88	110	0.88	0.37
12	2.32	4.85	22	1.78	2.45	40	1.37	1.24	56	1.18	0.84	120	0.85	0.34
13	2.24	4.45	24	1.71	2.25	....	....	....	....	....	....	....	....	....

the variation for other rainfall curves and states of areal development to be secured by applying to the basic values a runoff factor (Fig. 13) based on their volume of flow for the intercept of clock time containing the intense runoff to gutter.

Construction of the Runoff-in-Conduit Hydrograph.—The drainage area for the computation is rectangular in shape with the unit areas consisting of five-acre blocks, 330 ft by 660 ft, with the long side north and south, drainage flow south and east. The trunk drain (13,860 ft long) flows south along the east side of the drainage area and intercepts a lateral (4,290 ft long) every 660 ft. Each lateral serves 14 blocks. The rectangular shape, although common in urban drainage, is a special condition of shape; a method of correction of time of concentration for variation of shape is developed subsequently herein (see Fig. 14).

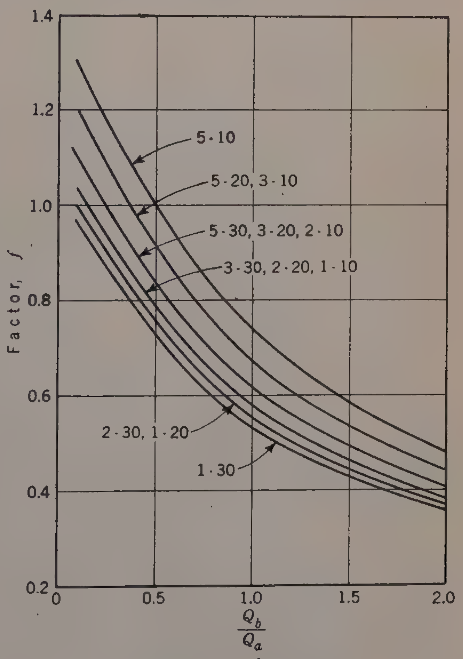


FIG. 14.—CHART FOR COMPUTATION OF POINTS OF CONFLUENCE

The inlet time of the initial block is 5 min. The mean velocity of flow in the conduit (V), based on full wetted perimeter, is assumed as 5.5 ft per sec. A number of tabulations of conduit storage capacity using different velocities of flow showed the same depth of storage capacity, when spread over the drainage area, for any given time of concentration.

The factors assumed for computations are peak runoff rates (Table 7), lag time (Fig. 9(*d*)), time duration of 90% peak (Fig. 9(*c*)), and time of drawdown (Fig. 9(*b*)). The curves on these charts indicate the final values obtained from these studies, determined after numerous trial computations.

The steps of computation and construction are:

1. Plot the 5-min hydrograph from the initial block as a rate curve and as a mass runoff curve by altering the net rainfall curve for detention on the surface and in the gutters.
2. Plot a trial 10-min hydrograph as a rate curve.
3. At selected points on the clock-time scale, compute the cross-sectional area of conduit for the 5-min and 10-min hydrographs and calculate ratios thereof to full cross-sectional areas; the average of these ratios multiplied by the volume capacity of the conduits between 5-min and 10-min times of concentration equals the volume of water in the conduits at each selected clock time. These volumes (in cubic feet) divided by the square feet in the drainage area having the 10-min time of concentration yield depths of conduit storage in feet over the drainage area.
4. Plot the depth of conduit storage (in inches over the drainage area) below the mass runoff curve for the 5-min hydrograph for the clock time at the left side of the chart; beginning at this plotted point, plot the 10-min hydrograph as a mass curve.
5. Plot the remaining depths of conduit storage above the mass runoff curve for the 10-min hydrograph; if these points coincide with the mass runoff curve for the 5-min hydrograph, the assumed 10-min hydrograph is correct; if not, plot another 10-min trial hydrograph and repeat the operations.
6. Having derived the correct 10-min hydrograph, assume a 20-min hydrograph, and repeat the operations, computing volume of conduit storage between 5-min and 10-min times of concentration, and between 10-min and 20-min times of concentration.
7. Repeat the operation for 10-min increments in time of concentration.

For illustrating the method, the 20-min runoff-in-conduit hydrograph is constructed using computed data in Tables 8, 9, and 10, and the graphs in Fig. 15. Tables 8 and 9 contain computations for the storage capacity of conduits lying upstream from points having times of concentration between 5 and 60 min. Table 10 is a multiple table with a common horizontal scale of clock times of points selected for computing volumes of instantaneous conduit storage. These values of conduit storage depth are plotted in Fig. 15 above the 20-min mass runoff curve. These points have been balanced with the 5-min mass runoff curve to a reasonable but not mathematical accuracy.

The results of these analyses are summarized in the table of basic peak runoff rates (Table 7) and in a chart of the time elements of the basic hydrographs (Fig. 16). With these basic data and charts of runoff factors (Fig. 13) for various rainfall intensities and physical characteristics of the drainage area, peak runoff rates and hydrographs can be computed for use in design. In Fig. 16, the time of drawdown (100% to 20% of the peak runoff rate) is equal to the time of concentration, t_c, for any time exceeding 20 min. A table for plotting the basic standard hydrographs for times of concentration of from 5 to 60 min is filed in the Engineering Societies Library.[1a] Examples are shown in Figs. 3 and 15.

TABLE 8.—VOLUME INCREMENTS OF CONDUIT DETENTION CAPACITY

Time of concentration, in min (t_c)	Area acres (A)	Length, in ft (L)	Record peak runoff rate, in in. per hr (q_c)	Factor $\left(\frac{A\,L}{V}\right)$	Volume increment, in cu ft $\left(\frac{A\,L\,q_c}{V}\right)$	Cumulative totals, in cu ft
5	5	330	3.40	300	....	
6	10	330	3.13	600	1,020	
7	15	330	2.93	900	1,878	
8	20	330	2.76	1,200	2,637	
9	25	330	2.63	1,500	3,312	
10	30	330	2.50	1,800	3,945	12,792
11	35	330	2.41	2,100	4,500	
12	40	330	2.32	2,400	5,061	
13	45	330	2.24	2,700	5,568	
14	50	330	2.17	3,000	6,048	
15	55	330	2.10	3,300	6,510	
16	60	330	2.05	3,600	6,930	
17	65	330	1.99	3,900	7,380	
18	70	660	1.94	8,400	7,761	62,550
20	140	660	1.85	16,800	16,296	78,846

TABLE 9.—AVAILABLE CONDUIT DETENTION BETWEEN STATED TIMES OF CONCENTRATION

Time of concentration, in min (t_c)	Area acres (A)	Unit[a]	TIMES OF CONCENTRATION						Total detention, in in.
			5–10 min	10–20 min	20–30 min	30–40 min	40–50 min	50–60 min	
10	30	MCF	12.8	....	....	....	....	....	0.117
		in.	0.117	....	....	....	....	....	
20	140	MCF	25.6	115.8	....	....	....	....	0.278
		in.	0.050	0.228	....	....	....	....	
30	490	MCF	89.5	364.6	283.3	....	....	....	0.413
		in.	0.050	0.204	0.159	....	....	....	
40	840	MCF	153.5	613.4	283.3	553.2	....	....	0.525
		in.	0.050	0.201	0.093	0.181	....	....	
50	1,190	MCF	217.5	862.2	283.3	553.2	771.7	....	0.623
		in.	0.050	0.200	0.066	0.128	0.179	....	
60	1,540	MCF	281.5	1,111.0	283.3	553.2	771.7	957.7	0.708
		in.	0.050	0.199	0.051	0.099	0.138	0.171	

[a] MCF = 1,000 cu ft.

Runoff Factors.—The derivation of runoff factors (F_{ro}) is based on the volume of mass runoff-to-gutter curve between 80 and 120 min clock time shown as intercepts for various hourly rates of rainfall in Fig. 17, which is constructed as follows: Using the standard pattern of mass rainfall curve with hourly intensities varying from 0.7 to 6 in. per hr, net runoff-to-gutter hydrographs are computed for sand, loam, clay, and impervious surfaces with loss rates for $M = 1$ from Fig. 4,[6] and with depression storage losses of 0.20, 0.15, and 0.10 in. for sand, loam, and clay.

TABLE 10.—COMPUTATION OF RUNOFF HYDROGRAPH WITH CONDUIT DETENTION

t_c	q_c	CLOCK TIME, IN MINUTES														
		60	70	80	85	90	93	96	100	105	110	120	130	140	150	160
(a) RUNOFF RATE AT CLOCK TIME																
5	3.40	0.40	0.51	0.72	0.90	1.40	3.39	1.58	1.04	0.70	0.50	0.28	0.19	0.16	0.16	0.16
10	2.50	0.40	0.50	0.67	0.80	1.06	2.46	2.44	1.20	0.73	0.45	0.56	0.25	0.19	0.16	0.16
20	1.85	0.40	0.49	0.65	0.79	1.00	1.44	1.83	1.83	1.25	0.77	0.36	0.28	0.22	0.19	0.16
(b) PERCENTAGE OF PEAK RUNOFF RATE AT CLOCK TIME																
5	3.40	11.8	15.0	21.2	26.5	41.2	99.7	46.5	30.6	20.6	14.7	8.2	5.6	4.7	4.7	4.7
10	2.50	16.0	20.0	26.8	32.0	42.4	98.5	97.7	48.0	29.2	22.4	14.0	10.0	7.6	6.4	6.4
20	1.85	21.6	26.5	35.2	42.7	54.1	77.9	98.9	98.9	67.5	41.6	19.5	15.1	11.9	10.3	8.6
(c) PERCENTAGE OF AREA AT CLOCK TIME																
5	....	17.8	21.6	27.6	32.1	43.7	87.3	47.6	35.3	26.9	21.3	13.8	10.2	8.8	8.8	8.8
10	....	22.8	26.2	32.4	36.3	44.3	85.9	85.4	48.7	34.2	29.0	20.6	15.4	13.0	11.3	11.3
20	....	28.0	32.2	39.2	44.7	52.5	70.0	86.4	86.4	62.3	43.8	25.8	21.8	17.8	15.7	14.3
(d) AVERAGE PERCENTAGE OF AREA AT CLOCK TIME																
5–10	....	20.3	23.9	30.0	34.2	44.0	86.6	66.5	42.0	30.5	25.1	17.2	12.8	10.9	10.1	10.1
10–20	....	25.4	29.2	35.8	40.5	48.4	77.9	85.9	67.5	48.2	36.4	23.2	18.6	15.4	13.5	12.8
(e) CONDUIT DETENTION IN USE AT CLOCK TIME, IN 1,000 CU FT																
5–10	25,584[a]	5.2	6.1	7.7	8.7	11.3	22.2	17.0	10.7	7.8	6.4	4.4	3.3	2.8	2.6	2.6
10–20	115,812[a]	29.4	33.8	41.5	46.9	56.0	90.2	99.5	78.2	55.8	42.2	26.9	21.5	17.8	15.6	14.8
Summation		34.6	39.9	49.2	55.6	67.3	112.4	116.5	88.9	63.6	48.6	31.3	24.8	20.6	18.2	17.4
(f) CONDUIT DETENTION IN USE AT CLOCK TIME, IN INCHES																
20	....	0.068	0.079	0.097	0.109	0.132	0.222	0.229	0.175	0.125	0.096	0.062	0.049	0.041	0.036	0.034

[a] Total D_c available between times of concentration, in cubic feet.

Referring to Table 6, the rainfall on the pervious area is that falling directly upon it, plus 90% of that falling on impervious area and running on to pervious area. The runoff rate from pervious areas, which varies as $M^{0.59}$, is different for the different isohyetals (see Table 5 and Fig. 2). A sample computation of the runoff factor for an area in level residence classification, loam soil, on isohyetal 1.77, for an hourly rainfall rate of 1.77 × 10-yr factor 0.762 (Fig. 2) or 1.35 is as follows: Pervious area rainfall = 1.45 (Table 6) × 1.35 = 1.96, the converted hourly rate; intercept (Fig. 17) (hourly rate 1.35) for impervious area = 0.97; intercept (Fig. 17) (converted hourly rate 1.96) for pervious area = 1.02; and runoff factor

$$= \frac{(A_d \times \text{impervious intercept}) + (A_p \times M^{0.59} \times \text{pervious intercept})}{\text{Impervious intercept for basic hourly rate}}$$

$$= \frac{(0.40 \times 0.97) + (0.40 \times 0.755 \times 1.02)}{0.71} = 0.98 \text{ (see Fig. 13 for 40-2).}$$

The impervious intercept (0.71) for the basic hourly rainfall rate, 1.01 (10-yr B curve, Fig. 1) is used as a divisor in the foregoing equation for the runoff factor to make the results applicable to the basic runoff rates (Table 7). Runoff factors for various drainage area classifications are shown in Fig. 13.

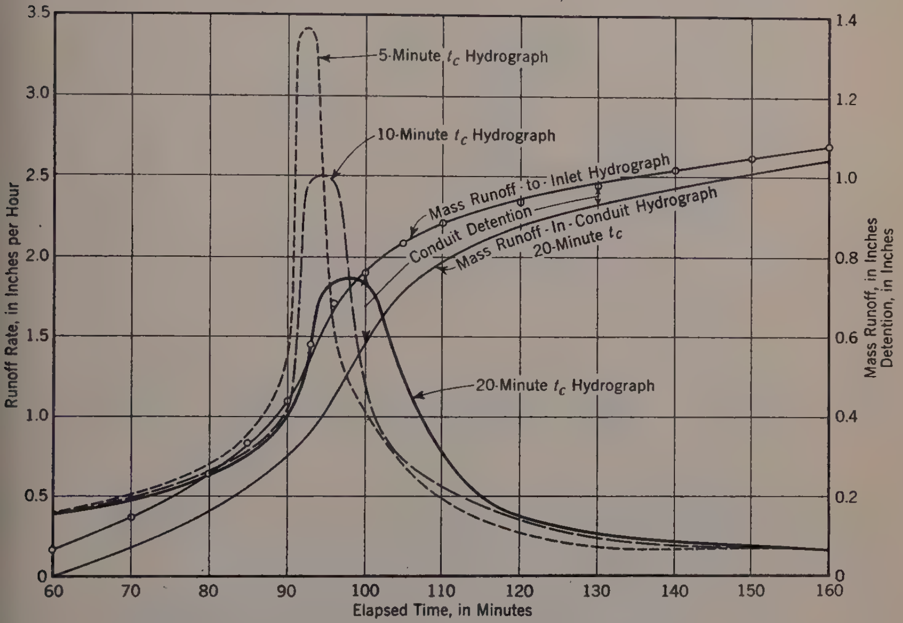


FIG. 15.—BASIC HYDROGRAPH FOR RUNOFF IN CONDUIT

A complete set of charts for the classifications listed in Table 6 has been filed with the Engineering Societies Library. The product of the runoff factor for a given area and the basic runoff rate for the time of concentration is the rate of runoff (q_c) in cubic feet per second per acre.

The values 1.12 and 2.65 in Fig. 13 indicate the 50-yr isohyetal on which the drainage area is located (see Fig. 2).

VII. HYDRAULICS OF THE PROCESS OF SUMMING HYDROGRAPHS

The process of summing hydrographs involves a hydrograph at point A on a drain being routed through a reach of conduit to point B where it is combined with a hydrograph from an incoming lateral and flows on to another junction point C, and so on.

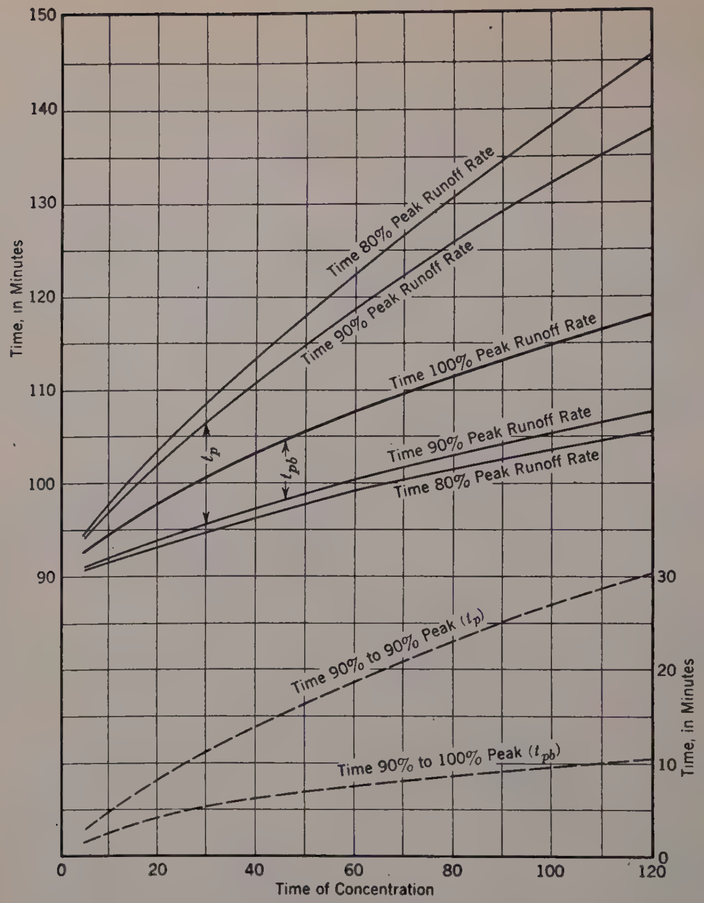


FIG. 16.—TIME ELEMENTS OF THE HYDROGRAPH

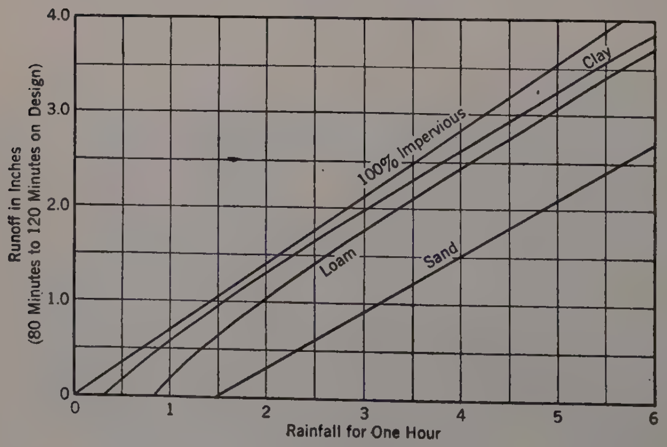


FIG. 17.—INTERCEPTS OF RUNOFF VOLUME, RUNOFF TO GUTTER CURVE

From studies of the basic hydrographs previously discussed, a method has been devised for transforming the rate hydrograph for the travel through a reach of conduit—first by a reduction of peak rate, and second by adjustment of the times of occurrence of various percentages of the new peak rate in the front and the back of the hydrograph.

Reduction of Peak Rate.—For this purpose, percentages of reduction of peak rate per minute of flow were computed for various times of concentration.

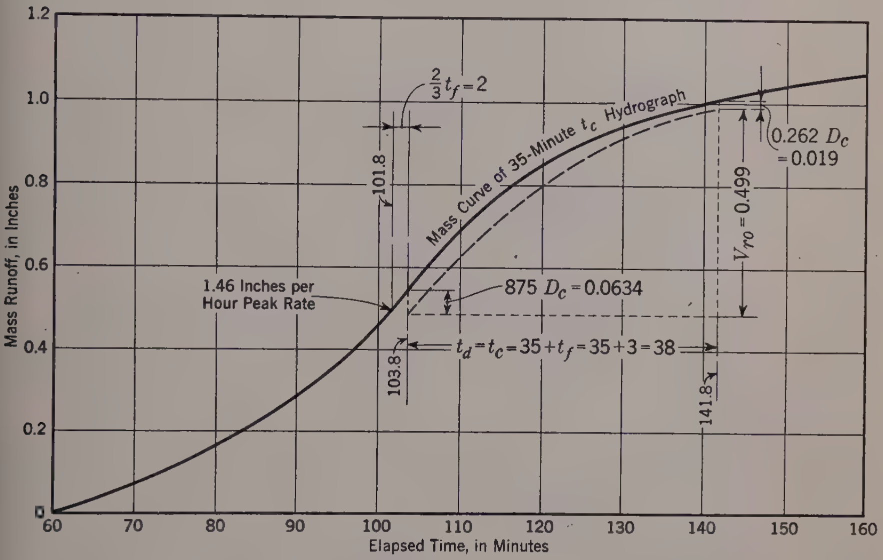


FIG. 18.—COMPUTATION OF REDUCTION FACTORS FOR CONDUIT DETENTION

In Fig. 18 the time of flow (t_f), in minutes, is based on the mean velocity of the conduit flowing full, the steps in the computation being as follows:

1. The base hydrograph of any time of concentration (say, 35 min) is drawn as a mass curve with time of peak 101.8 min (Fig. 16).
2. The time position of the reduced peak is set to the right of the original peak by two thirds the time of flow (for a wave velocity 1.5 times the mean full-flow velocity) or at 103.8 min; the position of 20% peak is at 141.8 min or 103.8 min plus the time of drawdown (original t_c plus 3 min t_f).
3. The volume of available added conduit detention, in inches depth on the drainage area (D_c), is $(t_f \times 60 \times V)\left(q_c \frac{A}{V}\right)\left(\frac{12}{43{,}560\ A}\right) = 0.0165\ t_f\ q_c = 0.0724$ in. This volume, the product of the length of conduit and its cross-sectional area, is independent of both the velocity of flow and the drainage area.
4. As the flow in the drain is ordinarily free flow, the approximate added detention for peak flow is 87.5% of capacity or 0.0634 in., and for 20% peak, 26% or 0.0190 in.
5. Using $d_o = 0.499$, $t_d = 38$ min, and K for a t_c of 38 min $= 106.9$ in Eq. 4, a value for q_c of 1.40 is obtained.

6. The reduction for 3 min time of flow is 4.3% or 1.43% per min. Based on a series of similar computations, the values in Table 7, Col. B, were obtained.

Travel Velocity of the Flood Wave.—The velocity of travel of the front and peak of a flood wave is higher than the mean velocity computed for the peak depth while the velocity of the recession side recedes to less than the same mean velocity.

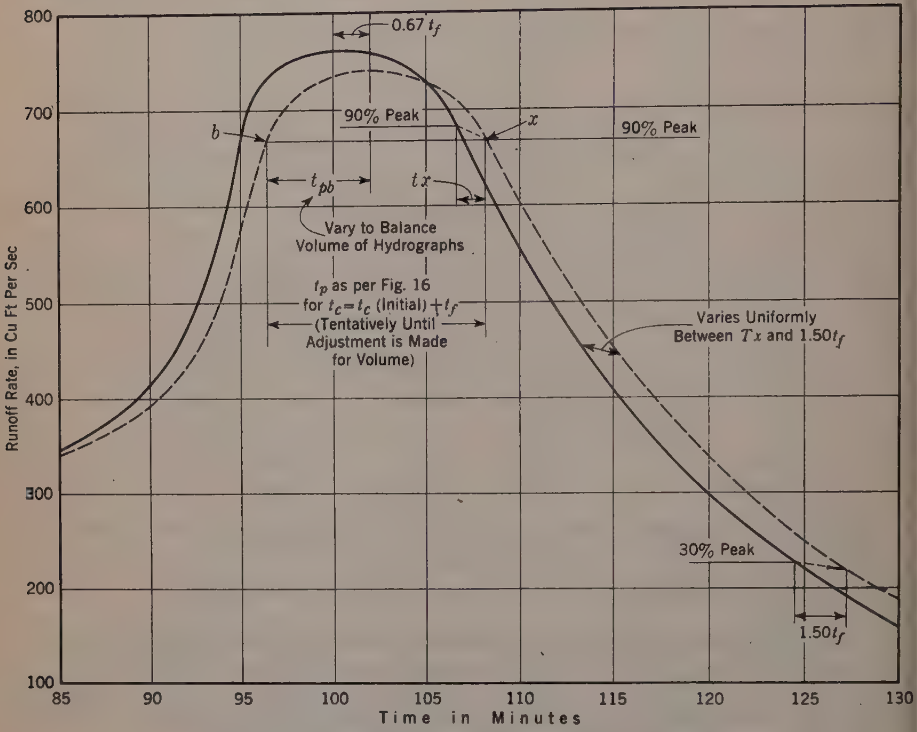


FIG. 19.—TRANSPOSITION OF A HYDROGRAPH FOR FLOW

An experiment along these lines by the Los Angeles County Flood Control District[11] was performed in an open rectangular channel 13,071 ft long between two reservoirs; the slope varied between 1.078% and 1.625%. Four releases of water were made, the last being in the form of a flood wave with a peak of 1,400 cu ft per sec. Gagings were made at five stations. In part the conclusions are:

> "7. For routing flood hydrographs down channels, a travel time based on a wave velocity affords a better approximation to true conditions than a travel time based on normal velocities. From the present test, the wave velocity ranges between 120% and 150% of the normal velocity for sudden changes of quantity."
>
> "11. The limitations of the data and conditions of flow must be kept in mind when applying these conclusions to other sets of conditions. * * *."

---

[11] "Field Test of Discharge from Puddingstone Dam and Flow in Puddingstone Diversion Channel," Los Angeles County Flood Control Dist. (unpublished).

In the experiment the normal velocity was based on open channel flow. If the percentages in conclusion 7 are expressed in terms of full-wetted perimeter, mean velocity, upon which time of flow is based in this paper, they become 140% to 175% (or 120% to 150% divided by 0.86).

From this experiment and a study of the time relations from rainfall-runoff records, the selection of 150% of mean full flow velocity of a closed conduit for the wave velocity of peak runoff rate at efficient free flow depth is reasonable and consistent with current engineering practice.

For the time movements of all phases of the hydrograph, the graph in Fig. 19 was developed from analyses of the time relations from rainfall-runoff records as recorded in Figs. 9 and 16.

Method of Summing Hydrographs.—This method is illustrated by an example. Two areas A and B have inlets 2 min time of flow apart. Area A has 700 acres of 50-2 development located on 50-yr isohyetal 2.22, $t_c = 40$ min; B has 300 acres of 40-2 on isohyetal 1.77, $t_c = 25$ min. A table filed in the Engineering Societies Library is used for plotting hydrographs.

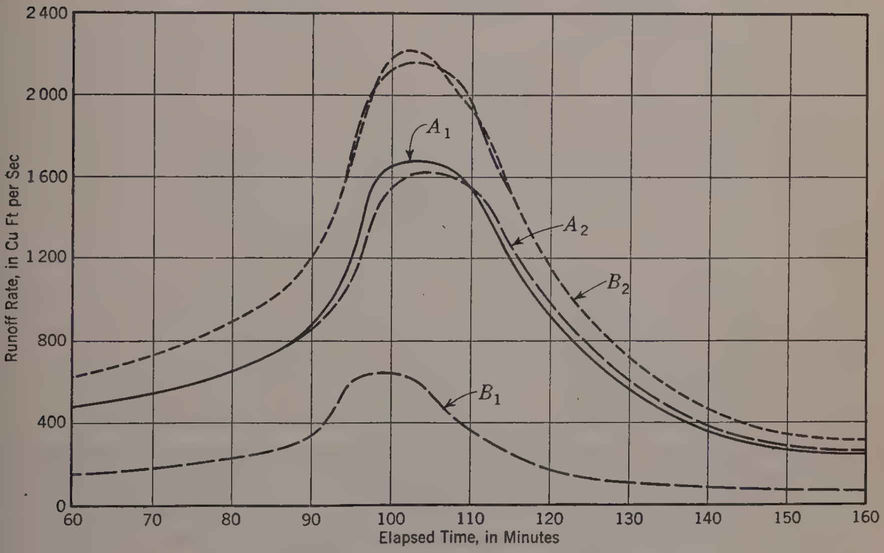


FIG. 20.—TRANSPOSITION AND SUMMATION OF HYDROGRAPH

For designing the conduit A to B, Q (peak runoff rate in cubic feet per second) = 700 acres × 1.80 (Fig. 13) × 1.37 (Table 7, Col. A) × 0.966 (Fig. 5) = 1,668 cu ft per sec. This quantity, applied to the 40-min basic hydrograph, yields hydrograph A_1 in Fig. 20.

For plotting hydrograph A_2 at the end of the reach A–B, the total reduction of peak for conduit detention is 2.45% (Table 7, Col. B), and the distribution factor becomes 0.954 because of the added 300 acres at point B; the peak rate for hydrograph $A_2 = 97.55\% \times \frac{0.954}{0.966} \times 1{,}668$ cu ft per sec = 1,607 cu ft per sec. Curve A_2 is plotted by the method of Fig. 19.

For plotting the hydrograph for area B, $Q = 300 \times 1.32$ (Fig. 13) $\times 1.68$ (Table 7) $\times 0.954$ (Fig. 5) $= 634$ cu ft per sec. The distribution factor is based on the combined areas A and B. Curve B_1 is the result.

Curves A_2 and B_1 are summed to form the tentative curve B_2. Because the mechanical process of summing does not evaluate the effects of hydraulic disturbances of the confluence, correction above 80% peak is made with the aid of Fig. 16:

1. By inspection of the uncorrected hydrograph at the points of 90% and 80% peak, select by trial a time position and value of 100% peak. Reconstruct the curve between points of 80% peak.
2. Check the revised curve against the original curve for agreement in volume between points of correction. The final value of Q for B_2 is 2,150 cu ft per sec.

Variation of Time of Concentration.—In summing hydrographs it is found frequently that the resulting hydrograph has time and peak characteristics materially different from those of either of the incoming hydrographs.

The problem was analyzed by summing a trunk-line hydrograph (the line with the larger time of concentration, t_{ca}) and a lateral hydrograph (t_{cb}) for a large number of combinations of times of concentration, peak runoff rates, and times of flow from the point of concentration on the trunk next preceding the junction. By trial, standard hydrographs were found to fit the summation of the hydrographs and the results were tabulated. From this tabulation the chart in Fig. 14 was constructed in which (below the junction):

$$t_{cx} = t_{cb} + f\,(t_{ca} - t_{cb}) \qquad (6)$$

To avoid undue influence of small increments of flow on the major trunk flow, a value of $f = 1$ is used when $\frac{Q_b}{Q_a}$ is less than 0.1. For $t_{ca} - t_{cb} = 40$ min, $f = 1$ when $\frac{Q_b}{Q_a}$ is less than 0.2; for 50 min, $f = 1$ when $\frac{Q_b}{Q_a}$ is less than 0.1. The legends 2-10, etc., on the curves mean a 2-min time of flow on the trunk from the next preceding point of concentration above the junction, and a 10-min value of $t_{ca} - t_{cb}$.

Fig. 14 is used in the Peak Rate Method described subsequently and has been given as wide a range as practicable because of the greater speed of operation of that method.

VIII. METHODS AND FORM OF RUNOFF COMPUTATION

Use of the Method of Summing Hydrographs described previously is confined to cases in which the Peak Rate Method is not applicable.

Peak Rate Method.—This method is subject to the following conditions:

1. When the hourly intensity of rainfall for the selected frequency is not constant over the entire drainage area (see Fig. 2), the hourly intensity for each point of concentration is computed by weighting the intensities within the drainage area by the effective areas to which they apply.
2. When a lateral and a trunk line combine with times of concentration and peak rates of runoff falling outside the limitations imposed by the notes

in the text, following Eq. 6, the Method of Summing Hydrographs is used for the continuation of the trunk line; the laterals to the continuation of the trunk are designed by the Peak Rate Method.

3. The outfall from a retarding reservoir is designed by the Method of Summing Hydrographs, after the inflow to the reservoir has been routed through by use of the reservoir formula (inflow = outflow plus storage).

4. Other limitations may be imposed by the judgment of the designer.

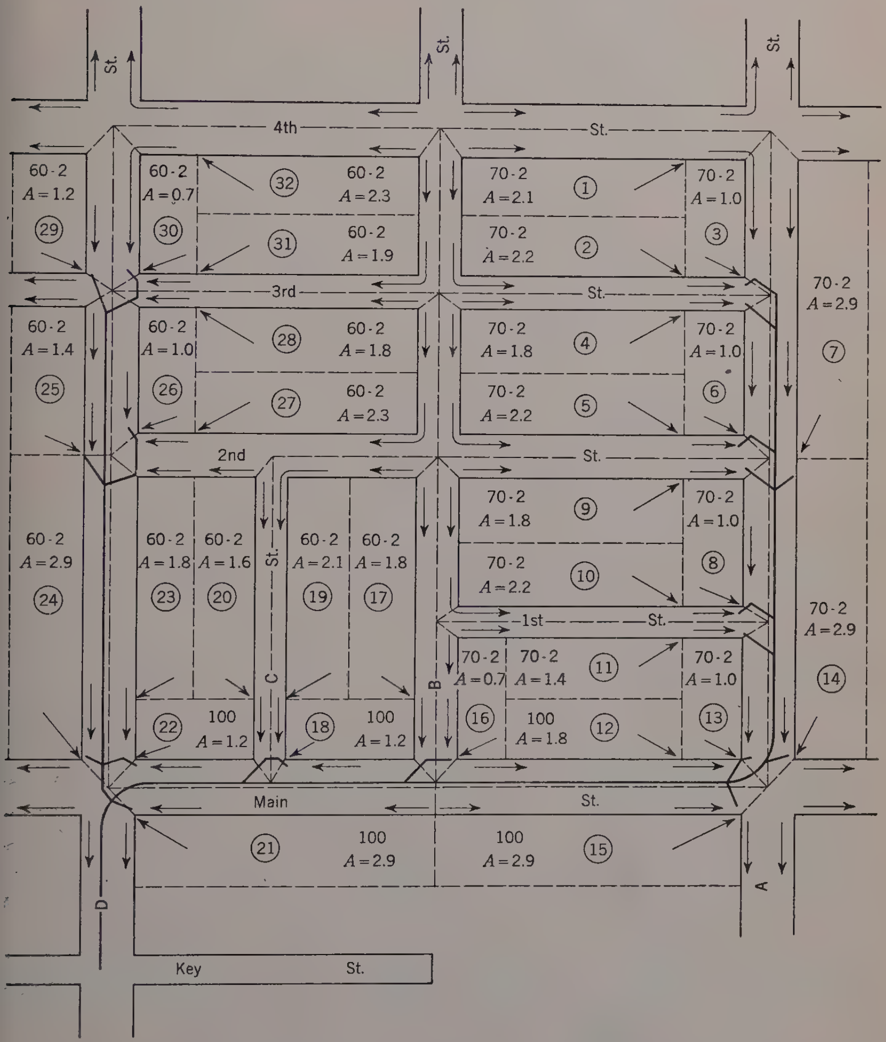


FIG. 21.—DRAINAGE AREA, PEAK RATE METHOD

Tables 6 and 7 (a table of inlet times[1a]), and Figs. 2, 5, 13, and 14 are used in this method.

A sample computation for the drainage area in Fig. 21 is shown in Table 11, which is largely self-explanatory:

TABLE 11.—PEAK RATE METHOD APPLIED TO MAIN STREE

No.	DRAINAGE AREA No. (1)	Acres (2)	Symbol (3)	CLASSIFICATION, AREAS AND RUNOFF QUANTITIES 100 (4)	70–2 (5)	60–2 (6)	(7)	(8)	(9)	(10)	(11)	1-hr rainfall rate (12)	Distribution factor (13)
													MAI
1 2	1–2–3–4	7.1	A ΣA q_e Q		7.1 7.1 2.75 19.5							1.01	1.00
3 4	5–6–7–9	7.9	A ΣA q_e Q		7.9 15.0 2.60 39.0							1.01	1.00
5 6	8–10–11	4.6	A ΣA q_e Q		4.6 19.6 2.49 48.8							1.01	1.00
7 8	12–13–14–15	8.6	A ΣA q_e Q	4.7 4.7 2.76 13.0	3.9 23.5 2.40 56.4							1.01	1.00
9 10	16–17	2.5	A ΣA q_e Q	 4.7 2.62 12.3	0.7 24.2 2.28 55.2	1.8 1.8 2.10 3.8						1.01	1.00
11 12	18–19–20	4.9	A ΣA q_e Q	1.2 5.9 2.54 15.0	 24.2 2.21 53.5	3.7 5.5 2.03 11.2						1.01	1.00
13 14	21 and D St. lateral	21.4	A ΣA q_e Q	4.1 10.0 2.49 24.9	 24.2 2.17 52.5	17.3 22.8 2.00 45.6						1.01	1.00
15 16			A ΣA q_e Q										
													D S
17 18	28–29–30–31–32	7.9	A ΣA q_e Q			7.9 7.9 2.37 18.7						1.01	1.00
19 20	25–26–27	4.7	A ΣA q_e Q			4.7 12.6 2.26 28.5						1.01	1.00
21 22	22–23–24	5.9	A ΣA q_e Q	1.2 1.2 2.6[illegible] 3.2		4.7 17.3 2.13 36.8						1.01	1.00
23 24			A ΣA q_e Q										

ND D STREET STORM DRAIN (10-YR DESIGN CURVE)

Amended 1-hr rain-fall rate (14)	ΣQ (15)	Friction slope, S (16)	$A\ C\ \sqrt{r}$ (17)	Pipe size (in.) (18)	Pipe length (ft) (19)	V (20)	t_c (21)	Factor f (22)	Station (23)	Storm drain locations (24)	No.
INE											
											1
1.01	19.5	0.01	195	24	375	7.1	6.0		36+15	A St. & 3d St.	2
							0.9				3
1.01	39.0	0.01	390	30	335	8.4	6.9		32+40	A St. & 2d St.	4
							0.7				5
1.01	48.8	0.01	488	33	325	9.0	7.6		29+05	A St. & 1st St.	6
							0.6				7
1.01	69.4	0.01	694	36	650	9.5	8.2		25+80	Main St. & A St.	8
							1.1				9
1.01	71.3	0.01	713	36	320	9.5	9.3		19+30	Main St. & B St.	10
							0.6				11
1.01	79.7	0.01	797	39	285	10.1	9.9		16+10	Main St. & C St.	12
							0.5				13
1.01	123.0	0.01	1,230	45	325	11.1	(10.4) 10.2	0.81	13+25	Main St. & D St.	14
							0.5				15
							10.7		10+00	D St. & Key St.	16
TERAL											
											17
1.01	18.7	0.01	187	24	340	7.1	7.0		9+80	D St. & 3d St.	18
							0.8				19
1.01	28.5	0.01	285	27	570	7.8	7.8		6+40	D St. & 2d St.	20
							1.2				21
1.01	40.0	0.01	400	30	70	8.4	9.0		0+70	D St. & N/S Main St.	22
							0.1				23
	40.0	....	....	....	....	....	9.1		0+00 = 13+25 M.L.	....	24

1. List the areas at their respective points of concentration (Cols. 1, 2, and 24).
2. Subclassify the areas (Table 6 and Cols. 4 to 11, Table 11).
3. Select the 1-hr rainfall rate from Fig. 2, with respect to the isohyetal and frequency (isohyetal 1.33 × 10-yr frequency factor 0.762 = 1.01, Col. 12).
4. When the gross drainage area exceeds 100 acres, adjust the 1-hr rate by the distribution factor (Fig. 5 and Cols. 13 and 14).
5. Determine the runoff factors from Fig. 13 in relation to the adjusted 1-hr rainfall rate and apply them to the base runoff rate for the time of concentration in Table 7 to derive runoff rate per acre (q_c in Col. 3).
6. Follow through with the routine tabling computations.
7. When a lateral is added, the time of concentration is adjusted in accordance with Fig. 14 (Col. 22).

IX. COMPARISON OF COMPUTED RUNOFF WITH RECORD RUNOFF

The methods of computation outlined herein are based on rather complete analyses of gagings taken in and near Los Angeles. Other records have become available, but have not been analyzed. Instead, runoff factors (F_{ro}) have been determined for the rainfall data of both local and outside records, both of the intense storm producing the runoff and of the antecedent precipitation, and these runoff factors have been applied to the basic runoff rates of Table 7 to produce computed values of runoff to compare with the actual values.

The runoff factor is based on surveyed percentages of the specific drainage area, namely A_i, A_d, and A_p. A series of curves is necessary to reflect the varying percentage of pervious runoff, which varies as $M^{0.59}$. A set of curves of runoff factors for Eastern Avenue (Station 3) is shown in Fig. 22.

TABLE 12.—COMPARISON OF COMPUTED AND ACTUAL VALUES OF

No.	Date	Hour	Percentage of runoff	Time of concentration, in min (t_c)	Rainfall rate, in in. per hr (I)	Rainstorm shape factor (F)	Standard rainstorm shape factor (F_s)
1	1/ 5/35	3:44 a.m.	24.6	28.3	1.19	0.805[a]	0.767[a]
2	12/30/36	9:04 p.m.	34.2	33.4	0.68	0.425	0.485
3	3/15/37	7:36 p.m.	44.0	32.7	0.88	0.484	0.618
4	2/11/38	12:27 p.m.	30.4	32.2	0.59	0.451	0.410
5	2/28/38	7:08 a.m.	42.5	29.4	1.09	0.790	0.716
6	2/28/38	11:30 p.m.	56.0	33.3	0.77	0.497	0.549
7	3/ 2/38	8:32 a.m.	70.0	30.8	1.06	0.600	0.720
8	3/ 2/38	9:30 a.m.	69.0	37.5	0.64	0.416	0.485
9	3/ 2/38	3:54 p.m.	73.0	37.1	0.55	0.430	0.415
10	12/18/38	7:43 p.m.	....	32.4	0.61	0.400	0.429
11	1/ 5/39	12:12 p.m.	....	30.0	0.73	0.403	0.486
12	1/ 7/40	7:53 p.m.	....	30.2	0.63	0.440	0.422
13	2/29/40	3:15 a.m.	....	31.0	0.64	0.405	0.435
14	3/31/40	4:47 p.m.	....	33.7	0.50	0.353	0.357

[a] Factors F and F_s are probably above average for area.

A set of computations for Station 3 is shown in Table 12, in which $F_s \doteq$ the standard rainstorm shape factor, which is the shape factor, F, for a standard shaped storm having a rainfall rate equal to that of the record storm for the specific time of concentration; F_s may be obtained by dividing the record intensity of rainfall (I) for this time of concentration by the rate for the same time duration on the standard 10-yr B curve (Fig. 1); I_H = the hourly rate of rainfall on the standard shaped rainfall curve corresponding to the rate for the specific time of concentration used, 1.01 $(F_s \times F_s)^{0.5}$, modified by the shape factor F, or 1.01 $(F \times F_s)^{0.5}$. The other headings have been defined or are self-explanatory.

The physical characteristics of local urban areas, Stations 3, 4, 10, and 11, are listed in Table 1; for Blocks A, B, and C, St. Louis, Mo., as reported[12] by W. W. Horner, M. Am. Soc. C. E., and F. L. Flynt, Assoc. M. Am. Soc. C. E., in Table 13. Record data, computations, and derived values of runoff, except for Station 3 (Table 12), are shown in tables filed in the Engineering Societies Library.[1a]

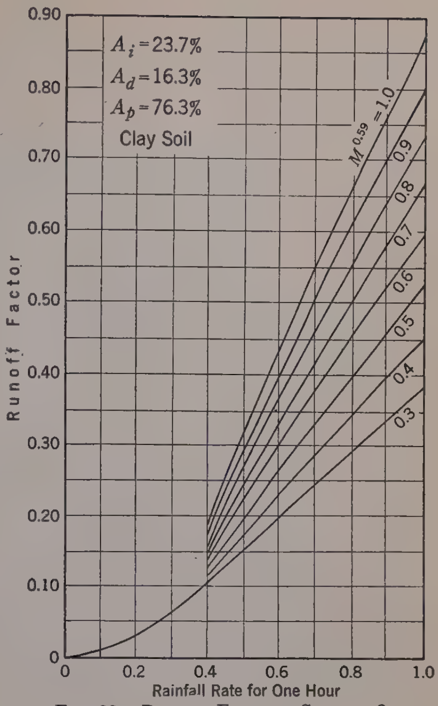


FIG. 22.—RUNOFF FACTORS, STATION 3

Record values of peak runoff rate for Stations 3 and 4 and for Blocks A and C have been corrected for overlap of preceding storms on the major storm (q_3). The hydrographs of the other areas were too irregular to justify this correction. In addition, the area for Station 4 has an inadequate lateral system

PEAK RUNOFF RATE; STATION 3, LOS ANGELES, CALIF.

Hourly rainfall rate $I_H = 1.01 \times (F \times F_s)^{0.5}$	$M^{0.59}$ (M = moisture factor)	Runoff factor (F_{ro})	Record peak runoff rate, in in. per hr (q_1)	q_1 corrected for prior storm (q_3)	Computed peak runoff rate, in in. per hr (q_c)	No.
0.793	0.44	0.36	0.430	0.413	0.57[b]	1
0.459	0.63	0.19	0.320	0.300	0.28	2
0.550	0.56	0.25	0.456	0.422	0.38	3
0.435	0.59	0.16	0.315	0.275	0.24	4
0.760	0.57	0.39	0.610	0.570	0.61	5
0.528	0.73	0.28	0.560	0.510	0.41	6
0.664	0.90	0.47	0.830	0.730	0.72	7
0.455	0.95	0.25	0.505	0.455	0.35	8
0.427	1.06	0.23	0.530	0.460	0.32	9
0.418	0.62	0.16	0.192	0.182	0.24	10
0.447	0.41	0.15	0.229	0.219	0.23	11
0.436	0.43	0.13	0.227	0.201	0.20	12
0.424	0.50	0.14	0.260	0.250	0.21	13
0.359	0.42	0.09	0.147	0.147	0.13	14

[b] One automatic rain gage only.

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[12] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140.

which causes overflow into street channels leading to the gaging system at about 0.45 in. per hr runoff rate, and causes overflow into another drainage area at about 0.60 in. per hr; this condition distorts computed corrections of the peak runoff rate.

TABLE 13.—CHARACTERISTICS OF URBAN DRAINAGE AREAS, ST. LOUIS, MO.

Station	Location	Area (acres)	Soil type	IMPROVEMENT (%)			Channel type
				A_i	A_d	A_p	
A	Belt and Ridge Avenues	2.3 to 1923 2.2 after	Clay	42.0 49.0	34.0^a 38.0^a	58.0 51.0	Gutter
B	City block No. 4841	3.27	Clay and loam	30.0	21.8^a	70.0	Gutter
C	Ewing and Washington Avenues	4.34	Clay	72.0	60.0^a	28.0	Gutter, pipe, egg-shaped sewer

^{a} Estimated.

Even with these deficiencies in the records, the comparison of the corrected record values of peak runoff rate with computed values (Fig. 12) shows a majority of the points lying within the lines of 20% variation. The mean of the points indicates that the computed values are a little below actual, especially when the correction for the preceding storm is considered.

CONCLUSIONS

1. The peak runoff rate for a given storm pattern is proportional to the volume of runoff resulting from the intense portion of the storm. The volume of runoff may be predicted—

   (*a*) For pervious areas, by infiltration tests on small soil plots and by analyses of rainfall-runoff records from partly or fully pervious drainage areas;

   (*b*) For impervious areas, by similar tests on impervious areas.

2. Rainfall-runoff records in which the high intensities of rainfall occur at or near the end of the storm (delayed pattern) disclose hydraulic phenomena and time relations which, when secured from drainage areas of various sizes and physical characteristics, enable the investigator to predict the approximate intensity and pattern of the runoff hydrograph that will occur under selected conditions of drainage area and rainfall.

When the method of computation derived from the delayed pattern storms is applied to the peak runoff rates for advance pattern storms, the approximate agreement obtained indicates that the same phenomena are present but are obscured by the runoff resulting from rainfall occurring after the peak of runoff.

3. The runoff hydrograph is the result of the travel and confluence of hydrographs from small units of area. As long as the urban drainage area is homogeneous in development and conforms to the topography of the natural drainage valley or swale, the hydrograph has a characteristic pattern of

smoothly varying rates; when hydrographs from dissimilar areas combine at a point of confluence, the outflow hydrograph has an irregular shape and a peak intensity unpredictable by a simple computation involving the size and physical characteristics of the drainage area and the intensity of rainfall. In such cases, the outflow hydrograph may be constructed by summing the hydrographs from each of the dissimilar areas.

4. Further accumulation of data is needed, emphasizing runoff records defining all stages of flow, accurate time correlation between rainfall and runoff, and experimentation on the storm flood wave.

5. For urban areas, in both Los Angeles and St. Louis, a comparison of actual runoff rates with those computed by this method from rainfall records (Fig. 12) shows most of the points falling within a 20% tolerance; agreement of points within the design range (above 1 cu ft per sec per acre) is more nearly perfect.

ACKNOWLEDGMENT

This investigation was conducted by the Bureau of Engineering, Department of Public Works, Los Angeles, under the direction of L. W. Armstrong, M. Am. Soc. C. E., principal civil engineer. Lloyd Aldrich, M. Am. Soc. C. E., is city engineer, and Merrill Butler, M. Am. Soc. C. E., is assistant city engineer (design). Prior to 1932, the work was supervised by F. L. Boyle, assistant civil engineer, and G. S. Tapley, Assoc. M. Am. Soc. C. E., civil engineering associate. At that time, the writer undertook the work, with H. W. Fraim, civil engineer, who supervised field and laboratory activities. Analyses and computations were performed by H. B. Brown and H. P. Larsen, civil engineering associates. Operation of automatic recording instruments was by F. J. Doran and A. A. Weston, civil engineering associates.

The Los Angeles County Flood Control District made its voluminous records and analyses available. Maxwell F. Burke, Assoc. M. Am. Soc. C. E., of that organization was very helpful in discussing the many phases of the problem.

The Water Department and the Engineering Bureau of the City of Pasadena cooperated in gaging rainfall and runoff in that city and by making available other records collected by them independently.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

HIGHWAY ENGINEERING EDUCATION

PROGRESS REPORT OF THE COMMITTEE OF THE HIGHWAY DIVISION

The Committee on Highway Engineering Education was formed at the direction of the Executive Committee of the Highway Division to investigate the general status and the adequacy of instruction in highway engineering in the colleges or universities of the United States, with a view to recommendations for expansion or improvement, if found necessary or desirable.

Procedure

As originally organized, this Committee comprised a representative from each Zone of the Society, with a Chairman and Vice-Chairman. After a preliminary discussion, a questionnaire was drawn up for distribution to engineering schools to elicit the desired information. Three of the Zone members used an identical questionnaire, and the fourth drew up one differing in form but agreeing in substance. Committee members were authorized to use their discretion in the distribution of these inquiries, with the result that in three Zones a representative list of institutions was selected by the respective members. In the fourth, the coverage of recognized engineering schools was nearly, if not quite, complete. Detailed replies were received from a total of sixty-one institutions in continental United States, one from Canada, and one from Hawaii. The form of questionnaire used in three Zones was as follows:

> "The Highway Division of the American Society of Civil Engineers, having formed a Special Committee to study the present status of Highway Engineering education, requests your cooperation in furnishing information concerning the scope of instruction in this branch of engineering in your institution; together with such additional information or expression of your views concerning instruction in this field as may assist the Committee in reaching its final conclusions. Bulletins descriptive of highway courses are also requested.
>
> (1) Is Highway Engineering instruction given:—
>   (*a*) As a requisite in conjunction with general Civil Engineering instruction?
>   (*b*) As an elective under Civil Engineering?
>   (*c*) As a separate course leading to a degree in Highway Engineering?

Note.—Please forward all comments on this Report directly to Chairman Julius Adler, 2001 Architects Bldg., 17th and Sansom Sts., Philadelphia Pa. Progress Reports are published in *Proceedings* only.

(2) What courses of study (including credit hours per week and number of semesters) are offered or required under the special head of Highway Engineering as distinguished from general Civil Engineering?

(3) Does class or field instruction in surveying include special reference to highway location or relocation? (If so, give nature and extent.)

(4) If not offered as a separate course, does instruction in design of structures include special reference to highway bridges and drainage structures? (If so, give nature and extent.)

(5) If not offered separately, is instruction in the economics of highway improvement given in conjunction with some other subject? (If so, give nature and extent.)

(6) Is testing laboratory equipment available and in regular use for tests of:
  (*a*) Bituminous highway materials?
  (*b*) Nonbituminous materials, such as soils, stone (for quality), Portland cement, fine and coarse aggregates for concrete vitrified brick? (Please indicate materials on which test instruction is given.)

(7) Are current specifications for highway construction used or referred to in connection with instruction in this subject?

(8) If not offered separately, is instruction in traffic engineering given in conjunction with some other subject? (Please indicate the nature and scope of instruction in traffic engineering.)

(9) Is either (*a*) collateral reading or (*b*) visits to material plants or construction projects included in required work?

(10) Have you any recommendations for the advancement or improvement of college instruction in the general field of Highway Engineering?

Where details of courses of instruction [see Item 2] are readily obtained from printed bulletins accompanying your reply, reference may be made to such bulletins."

Scope of Instruction

The first question in the inquiry was intended to determine the status of highway engineering instruction in a very general way. In the sixty-three schools from which replies were received, some highway engineering instruction is given as a requisite under the civil engineering course, but the amount and scope of such required instruction vary widely. At least twenty of these institutions are offering, in addition to required work in this field, an elective in the senior year for students who wish to specialize in highway work; but again there is considerable variation in these elective courses. No engineering school reported a course leading to a degree in highway engineering. The University of Michigan, at Ann Arbor, offers a course in transportation engineering (leading to a degree) in which the student may specialize in highway engineering. In the West and Southwest, specialized undergraduate curricula in highway engineering are offered by Oregon State College (Corvallis), University of Washington (Seattle), and University of Texas (Austin). A number of other institutions situated in, but not uniformly distributed over, the extensive area lying between the Appalachian Mountains and the Pacific Ocean are offering fairly broad required instruction or electives in highway engineering. In schools along the eastern seaboard, in general, the amount of instruction in highway engineering is appreciably less than that in institutions in other parts of the United States. Of those reporting in this area, only two colleges offer elective courses in the highway field in the senior year to supplement the required work.

The second question in the questionnaire form asked for an enumeration of specific courses (and credit hours) which fall in the special classification of highway engineering as distinguished from civil engineering in general. As previously stated, considerable variation was found in the amount of required highway instruction. In the reporting group as a whole, the total of specialized highway courses required for all civil engineering students (with a single exception) varied from a minimum of two hours for one semester to a maximum of ten semester credit hours. These figures do not include elective courses or required courses, such as route surveying, which may be partly devoted to the highway field.

Subsequent questions in the inquiry were designed to elicit more detailed information concerning the scope and character of instruction, the available equipment, and the field and laboratory work pertaining to the principal phases of highway engineering. The purpose of these questions and the status of instruction work as indicated by the answers follow.

Surveying

Because of the tremendous mileage of public roads in the United States, route surveying courses should differentiate between the principles and problems of highway location as distinguished from railroad location. Nearly all schools indicated their awareness of this as evidenced by the character of their field and classroom work in surveying courses.

Design of Structures

Obviously the number of instances of required construction or reconstruction of highway bridges (ranging from very large to quite small) far exceeds that of railroad bridges. Hence, a proper proportion of the time allotted to bridge design should be devoted to highway bridges, as compared to the former, nearly standard practice of dealing only with railroad loadings in design courses. Among the reporting schools, only one specifically excluded highway structures from design courses and one other failed to answer the question clearly. All others recognized the importance of highway structures in both steel and reinforced concrete design.

Highway Economics

The subject of highway economics is involved in, and should furnish the answer to, problems of alinement, gradient, width of right of way, and character and width of surface improvement. A wide variety of traffic, climatic, topographic, and soil conditions must be dealt with, and the proper selection of the surfacing material from those available within reasonable economic limits is in itself no mean problem.

The replies from the reporting schools indicate, on the average, a less satisfactory treatment of this phase of highway engineering than of the two previously enumerated. A few engineering schools have a required course in highway economics for all civil engineering students and others offer it as an elective to seniors or graduate students. Others include the subject to a varying degree of thoroughness either in a general course in highway engineering or in engineering economics. Many, however, do not indicate any real coverage of this important subject.

Testing Highway Materials

Highway materials include a wide variety, all subject to technical specification or test, some of which are of equal importance in other types of engineering structures. Among the more important are soils; stone, gravel, or slag as road metals; aggregates for use in concrete; highway concrete mixes (as distinguished from structural concrete mixes); asphalts, tars, and mixtures of these materials with mineral aggregates; and vitrified brick, granite block, etc.

A surprisingly high percentage of schools indicated the availability of testing equipment for both nonbituminous and bituminous materials (more frequently the former) either as part of their own equipment, or ownership with, or access to, the laboratories of state highway departments. Detailed information concerning the actual amount of instruction in, and performance of, these tests was not furnished in many cases, but, in a few, it was stated that student experience was limited to observation of demonstration tests, particularly of bituminous materials. Many replies indicated an active interest in soils tests, and an increase in testing equipment for this purpose. Although a few schools offered separate required courses in testing highway materials and others elective courses, under the more common arrangement, this subject was included as part of the general course in testing engineering materials.

Because of the specialized nature of this work, the wide variety of available materials, and the time required for laboratory testing, it would be unwise to advocate for the average student any great increase in the amount of time devoted to laboratory testing of highway materials. More advantageous results would be obtained from increased time used in the study of the general properties of these materials and the significance of the tests employed than from additional time spent in laboratory practice.

Current Practice and Progress in Highway Engineering

Under this general heading, two questions covered the classroom use of current standard specifications for highway construction—requirement of collateral reading, and inspection trips to construction projects and material plants.

Nearly all schools reported that they referred to, or made use of, current standard specifications for highway materials and highway construction, and approximately as many reported collateral reading and inspection trips as required (generally with written reports). A very few indicated their belief that time limitations did not permit collateral reading.

Highway Traffic Engineering

Over a period of years, increasing volumes of traffic in metropolitan areas and on main highways outside of cities and towns have served to direct attention to the necessity for developing scientific methods of regulating traffic flow. There is a growing recognition of the status of traffic regulation planning as an engineering problem rather than as a mere policing function. Even though, in many cases, highway engineers may not have jurisdiction over traffic regulation agencies, it is still a fact that competent highway design must include a knowledge of the principles of traffic regulation and must make suitable provision for it.

In view of the relative newness of this field of work, it is probably not surprising that about one half of the reporting schools stated that no instruction, or only passing reference, was given in traffic phases of highway engineering. In a number of other instances, the amount of instruction was limited to a very few periods included in one of the more general courses. More comprehensive instruction in traffic engineering was reported by not more than six schools, generally as a senior elective, and as graduate work in two cases.

Suggestions for Changes or Improvements in Highway Instruction

The final question in the questionnaire was directed at obtaining constructive recommendations from heads of departments or men engaged in highway engineering instruction. Many replies were received, covering a wide field; hence only those which were repeated several times will be mentioned in detail.

(*a*) The number of schools from which the opinion was expressed that highway engineering was not receiving the amount of time and attention in the school curriculum warranted by its relative importance in the field of engineering was slightly exceeded by the number expressing the view that sound engineering courses must concentrate on fundamentals and that they can devote but little time to specialties (in which class they placed highway engineering). A related viewpoint was that educational expansion should not go beyond the actual need for, or possibility of, absorption of students in a given field of work. Hence, although not recommending a reduction in their present (usually minimum) number of hours of instruction, these schools were opposed to any expansion. Most of the latter group are situated in the eastern part of the United States, with a very few in the central or western parts.

(*b*) Several replies stress the importance of a scientific approach in highway engineering instruction and criticize a tendency (presumably in textbooks) to concentrate more upon how things are done than why they are done in a given way.

(*c*) Several comments indicated the lack of suitable texts on some of the more specialized phases of the highway field, whereas others indicated a weakness due to a general tendency to cover too broad a field in a single volume, with the result that some important branches are not treated adequately. When more than one textbook is used, care must be taken to avoid the possibility of undesirable repetition.

(*d*) Many school administrators stress the importance of keeping abreast of current research and developments in the highway field—by maintaining an active contact with a state highway or other public engineering organization; by reference to current proceedings of technical organizations concerned with highways; and by occasional lectures by practicing engineers and good films dealing with the more important branches of highway construction.

(*e*) Student interest in highway engineering was reported generally as good. In practically all cases where it appeared to be lacking, the explanation was offered that an undue amount of political influence was affecting appointments and retarding advancement in this field of work, thus discouraging students from choosing elective highway courses. From a geographic standpoint, comments of this kind originated from an extensive area, indicating the existence of such situations in widely scattered states.

CONCLUSIONS

The Committee is disposed to avoid as completely as possible a controversial attitude in its review of the information placed before it. It is in complete sympathy with the viewpoint that engineering students must be graduated with a sound and thorough training in the basic fundamentals of engineering knowledge so that they may have a satisfactory foundation for beginning and progressing in any branch of their profession.

It does seem reasonable, however, to emphasize that the general field of highway construction and reconstruction, in spite of the great amount of work already done in the United States, is far from exhausted. For many years there was substantial concentration upon the task of improving arterial highways, which in many localities is far from completion. The further program of suitable improvement of secondary and less important highways (in a manner in keeping with their economic value) has been undertaken more recently, and is much less complete. In the meantime the enormous growth in interurban traffic is resulting in an increasing demand for the reconstruction of some of the older main highways, if not in all cases as superhighways—at least to a design that outmodes the standards adopted for principal highway construction in the active construction period which followed the first World War.

It is evident that time devoted to highway engineering in college curricula is influenced by conditions pertaining to the geographic location of the school. In New England and the Middle Atlantic States, where modern highway construction began at an earlier date than elsewhere, and where main highways have more nearly reached a state of general improvement, interest in highway engineering instruction, on the average, is at a lower ebb. In some localities more widely distributed, it appears that political influence over highway engineering is producing a similar result.

With all of the foregoing considerations in mind, the Committee has reached the following conclusions:

(1) Highway engineering is no more a specialty than sanitary engineering, railway engineering, or structural engineering. Its principles and practices find widespread application in both public and private work. As a required

subject in engineering curricula, it deserves sufficient instruction hours to cover the more important fundamentals of this branch of work.

(2) Required highway instruction in civil engineering courses could be fixed conservatively at four semester credit hours (or the equivalent) equally divided between: (*a*) Highway construction; and (*b*) highway design, economics, and administration. Regular courses in engineering materials should include the equivalent of one semester credit hour of laboratory and classroom work devoted to materials (including soils) which are used in highway work. Instruction in the design of concrete mixtures should distinguish between highway mixes and structural mixes.

(3) Apparently, highway surveying and location, and highway bridges and structures (with occasional exceptions), are receiving adequate instruction.

(4) In localities where conditions are favorable for the absorption of engineering graduates into the highway field on a reasonably attractive basis, supplemental instruction should be offered either through senior options or electives, or less frequently through graduate courses. No attempt will be made to define this supplemental work. The minimum instruction time previously recommended is insufficient to give detailed instruction in the subjects named and might not include reference to traffic engineering or highway transport. A number of local conditions may have a justified bearing upon the exact scope of such supplemental or advanced instruction.

(5) Instruction in highway engineering is as susceptible to a scientific approach as any other engineering branch that is not entirely mathematical. The properties of materials must be known and understood, but instruction in methods of design and construction should not be limited to a dry statement of facts. The method of approach should be based upon the reasons for the procedures adopted. Matter-of-fact specifications can be made interesting if the underlying reasons for the successive steps are sought and explained and, under this procedure, should leave a more lasting impression on the student's mind.

(6) Advanced instruction in highway engineering might well be given on something like a regional basis. The Committee does not advocate the unnecessary duplication of instruction by a considerable number of schools, but does believe that, if this matter were taken under consideration by the proper society or other organization, eventually there could be a proper geographical distribution of advanced courses in the general highway field in localities where the demand existed and conditions were favorable.

Respectfully submitted,
B. H. Wait, Zone I[1]
Roger L. Morrison, Zone III
Earl C. Thomas, Zone IV
T. Warren Allen, *Vice-Chairman*
Julius Adler, Zone II, *Chairman*
Committee on Highway Engineering Education

January 29, 1943.

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[1] Resigned from the Committee on February 9, 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DESIGN OF ST. GEORGES TIED ARCH SPAN

Discussion

BY J. M. GARRELTS, ASSOC. M. AM. SOC. C. E.

J. M. GARRELTS,[15] ASSOC. M. AM. SOC. C. E.[15a]—The relative economy of the tied arch form for this structure, compared with other possible types, is discussed by Professor Abbett. Professor Abbett made several economic studies of this kind in connection with this project and thus writes from experience. His discussion also emphasizes the attention given to the problem of appearance in the St. Georges design.

Mr. Karol has shown the close approximation of the value of H used in the preliminary design to the value resulting from the final computations. The difference is almost entirely accounted for by the effect of rib shortening and the extension of the tie girder. The values of H, obtained by a formula similar to Eq. 17 but with the last two terms of the denominator eliminated, or

$$H = \frac{\sum \frac{M^1\, m\, \Delta l}{I_{tr}}}{\sum \frac{M^2\, \Delta l}{I_{tr}}} \qquad (43)$$

are as follows:

Load at point:	Values of H
1	0.240
2	0.467
3	0.671
4	0.847
5	0.986
6	1.084
7	1.136

NOTE.—This paper by J. M. Garrelts, Assoc. M. Am. Soc. C. E., was published in December, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1942, by R. W. Abbett, M. Am. Soc. C. E.; April, 1942, by Jacob Karol, Esq.; May, 1942, by Alexander Dodge, Esq.; June, 1942, by Carlos A. Bejarano, Jun. Am. Soc. C. E.; September, 1942, by Messrs. A. A. Eremin, and A. M. Freudenthal; October, 1942, by C. H. Gronquist, Assoc. M. Am. Soc. C. E.; and November, 1942, by Harold E. Langley, Esq.

[15] Associate Prof., Civ. Eng., Columbia Univ., and Designing Engr., Waddell & Hardesty, New York, N. Y.

[15a] Received by the Secretary February 23, 1943.

The procedure, suggested by Mr. Karol, for proportioning the total moment between the arch rib and girder will be an aid in calculating the approximate stresses and in determining approximate sections that can be used as the basis for the final analysis.

Mr. Dodge first questions the procedure of neglecting the bending resistance of the arch rib in the preliminary design, although he offers no suggestion as to how the first or preliminary stresses can be computed. In designing any statically indeterminate structure, the designer either can make a guess as to the sections, can obtain them by comparison with a similar structure, or can apply an approximate stress analysis from which trial sections can be obtained. Then, a more accurate analysis is made and the sections are revised as necessary. This latter procedure was used in the design of the St. Georges bridge. The bending resistance of the arch rib later was included in the analysis, as explained in connection with Eqs. 10 to 17.

The objection of Mr. Dodge to the use of Eq. 10*a* may be likened to the objection which is sometimes raised against using $\frac{d^2y}{dx^2} = \pm \frac{M}{E\,I}$ for an originally straight beam, when the correct equation is

$$\frac{\dfrac{d^2y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} = \pm \frac{M}{E\,I} \qquad \text{(44)}$$

Using the reference cited by Mr. Dodge, values are found,[16] which result in Eq. 10*a*.

Mr. Dodge states that "A small initial curvature makes a great change in the effect of longitudinal forces on the deflection of the arch rib." The fact is that the change in curvature is essentially a function of the bending moment.

On the assumption of a parabolic rib and only five hangers, Mr. Dodge attempts to compute the forces in the structure. In the St. Georges bridge there are fourteen hangers and the rib is polygonal, being straight between hanger connections, as shown in Fig. 6. He contends that the writer's values for H are too high. The following values were determined from the model analysis made at Princeton University at Princeton, N. J., under the direction of Prof. E. K. Timby, Assoc. M. Am. Soc. C. E.—

Load at point:	Measurement of H at a point in member:	
	$L_7\,L'_7$	$U_7\,U'_7$
1	0.226	....
2	0.450	....
3	0.662	0.661
4	0.820	0.838
5	0.971	0.971
6	1.081	1.081
7	1.135	1.132

These values, which are in agreement with the theoretical values used, are probably more nearly correct than those computed by Mr. Dodge, using the assumptions he has made; and the increase in the moment M_T in Fig. 13 does not actually exist, if the writer's values of H are accepted as correct.

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[16] "Strength of Materials," by S. Timoshenko, D. Van Nostrand, 1930, Pt. II, pp. 468–469.

To summarize: Mr. Dodge's conclusions (1) and (2) are the result of a misconception of the writer's procedure in applying the theory used in the paper. Furthermore, his conclusions (3) and (4) appear to be based on a mistaken conception of the writer's development. His statement that "The great influence of longitudinal forces on the deflections of an arch rib must be emphasized" means nothing more than that the correct bending moment must be taken into account. If the deformations due to direct stresses are neglected, the deflection of an arch rib is almost entirely due to bending moment, and the analysis used by the writer certainly gives full attention to the moment. Study of the writer's development will show that there is no assumption that involves a horizontal displacement of the ends of the arch rib with respect to the tie girder.

Mr. Bejarano has kindly evaluated certain terms of Eq. 12 which the writer neglected in later formulas. From his result, the procedure seems to be justified. Although the effects of rib shortening and tie extension are small, they should probably be considered because a small change in H affects the moment by an amount equal to the change in H times y-y'.

The procedure of calculating the structure by transforming the tie to produce a rib with a transformed moment of inertia I'_{tr} also has been discussed by Mr. Bejarano. He shows that the general results by the two analyses would be almost the same but that in the details there are certain differences which indicate an advantage in assuming that action which is more nearly in accord with the structural behavior.

According to Mr. Eremin, the procedures used by the writer are not adequate for the design of the end connections of the arch rib to the girder. In the first place, the hanger $U_1 L_1$ has practically no change in length so that in triangle $U_1 L_1 L_0$ angle $U_1 L_0 L_1$ is changed only by the moment at U_1 and the moment at L_1. Since these two moments will have the same sense, that is, positive or negative, and both will be small, any resulting moment at L_0 also will be quite small.

The writer does not agree with Mr. Eremin's statement regarding the assumption of constant stiffening girder section in Eq. 17. Actually the summation in the equation indicates that the girder is considered made up of segments over each of which the section is constant. Actually fifteen sections, each one panel in length, were used in the analysis.

The design of the lateral system was made in a manner similar to that used for the lateral system of a curved-chord through-truss bridge. In the arch-rib lateral system, the arch rib forms the chords, and the laterals and struts, the web system, with a portal extending from U_2 to L_0. The top laterals and struts are determined in most cases on the basis of minimum $\frac{l}{r}$ rather than by stress.

The writer is grateful to Mr. Freudenthal for his discussion of the general deflection theory of arches and suspension bridges; also for calling attention to the effects of deflection on self-tied structures.

During the design of the St. Georges tied arch span, it was often spoken of as an inverted, self-anchored suspension bridge. Mr. Gronquist has pointed out this analogy. He has also called attention to the action of these two self-tied structures as contrasted with those externally anchored. In addition, Mr. Gronquist has discussed the use of the tied arch with a continuous girder and has outlined the procedure to be used in solving this type of structure.

The writer considered the development of Eq. 17 to be the same as that for Eq. 4, and therefore did not repeat it. The only difference in the two cases is that, in Eq. 17, the tie girder is considered as a transformed section which includes the bending resistance of the arch rib. Actually, in the assumed singly indeterminate structure, the statically determinate substitute structure can be selected in several different ways.

Mr. Langley's statement regarding formulas based on the deflection theory already seems to have been answered by Mr. Freudenthal and Mr. Gronquist. In this self-tied structure, the deflection theory agrees with the analysis given because the arch rib and tie girder deflect the same amount.

The buckling resistance of the arch rib was considered and was found to be ample; and, although the hangers are connected to the arch rib by rigid connections, the bending moment transmitted by the hangers is very small because the hangers are quite flexible.

In conclusion, the writer wishes to express his thanks to those who have discussed this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RELATIVE ANGULAR, LINEAR, AND TRAVERSE ACCURACIES IN CITY SURVEYS

Discussion

By S. A. Bauer, Assoc. M. Am. Soc. C. E.

S. Bauer,[12] Assoc. M. Am. Soc. C. E.[12a]—The writer is indebted to Colonel Coe; Professors Earnest, Kissam, and Rayner; and Messrs. Whitmore, Miner, and Byrd for their kind discussions of this paper. The discussions have been helpful in suggesting new approaches to this study of traverse accuracies.

Professor Rayner is correct that Eq. 5 (which attempts to view the over-all picture) tends to oversimplify the problem in relation to the individual traverse. Curves 4, 5, and 8, Fig. 5, offer a truer picture of the problem as applied to the individual traverse than that previously presented in Fig. 4, since the conditions within a specific traverse are more nearly met by curve 5 than by curve 6. In Fig. 5, curve 9 represents the value of the square root of the sum of the squares of the linear and angular errors (reduced to the common unit of one part in x thousands) at the various frequencies. In the normal ranges of accuracy, there is a marked agreement between calculated curve 9 and traverse curve 8. This would seem to indicate a compensating effect of the linear and angular errors within a traverse.

However, the linear errors referred to are the probable errors in the measurement of single lines, and the angular errors similarly are the probable errors in the observation of single angles. Curve 9 then might be the probability curve of accuracy for the setting of a single point by angle and distance as well as the curve for traverse accuracy. If so, then the number of sides, and therefore the length of a traverse, plays no part in the accuracy of closure.

This contradicts much that has been written, and it is particularly contradictory to Professor Rayner's 1928 study of the problem,[5] wherein the accuracy of traverse closure is given as a function of the length of traverse.

Note.—This paper by S. A. Bauer, Assoc. M. Am. Soc. C. E., was published in May, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1942, by Messrs. Cleveland B. Coe, W. H. Rayner, Philip Kissam, and George D. Whitmore, C. C. Miner and W. O. Byrd; and November, 1942, by G. Brooks Earnest, M. Am. Soc. C. E.

[12] (Bauer Surveys Co.), Cleveland, Ohio.

[12a] Received by the Secretary February 25, 1943.

[5] "Specifications for Transit Traversing and Stadia Leveling," by W. H. Rayner, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 679.

The writer finds no relationship of traverse accuracy to length of traverse but only to mean side length. Professor Rayner properly asks for explanation. On this point, further investigation has proceeded as follows: Seven cases of from two to six connecting traverse loops with the same general conditions were studied; in none of the seven cases was the over-all or outside loop closure better than the best smaller inside loop closures of which it consisted; in three cases, the outside loop closure was equal or inferior to the poorest inside loop closure; in seven cases, the outside loop closure was better than certain individual loops within itself; and, in eleven cases, the outside loop closure was inferior to certain individual loops within itself.

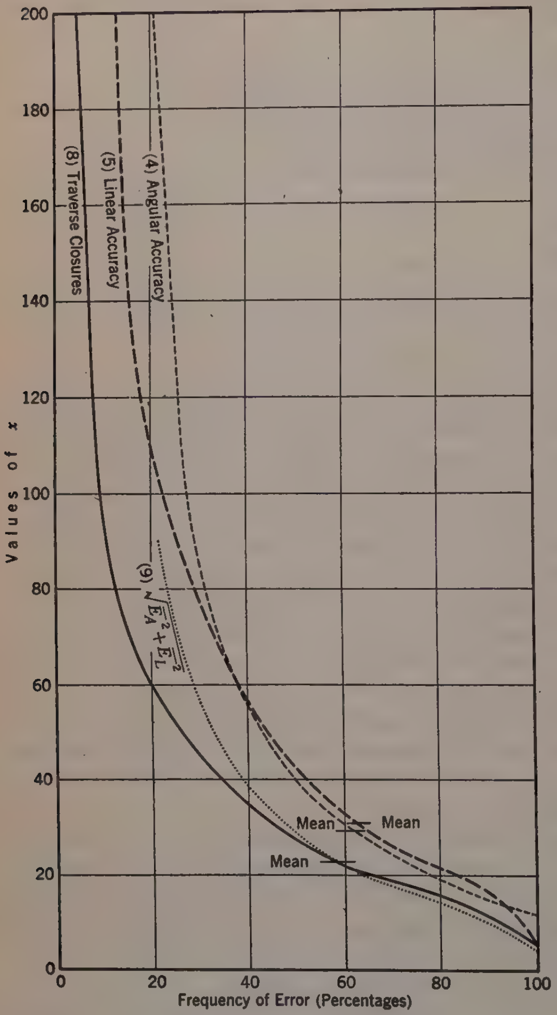

Fig. 5.—Traverse Measurement and Angle Accuracies (Expressed as One Part in x Thousands)

Six other cases of two or more interlocking traverses, having the same general topographic conditions but without an over-all outline closure, were examined. In two cases the longest loop of its group had the best closure of the group; four cases occurred in which the longest loop of a group had the poorest closure of the group; in eight cases, the longest loop had a better closure than some of the smaller loops within its group; and, in five cases, the longest loop of a group had poorer closures than others within the group.

The foregoing study of thirteen groups of traverses (each traverse being comparable to the others within its group as to terrain, side length, etc.) indicates no relationship between traverse closure and traverse length. This study would seem to indicate, also, that the compensation of errors does not extend throughout a traverse, that increased length of traverse will not help the closure, that the length of the traverse side, rather than the traverse length, affects the closure, that the compensation of errors occurs within a single side, but that in a traverse the side error becomes systematic.

The writer is not ready to state that this relationship extends to all instrumentation and procedures. These findings only can be attributed to the equipment and procedure used in this paper, since the limit of accuracy of the instrumentation may have been reached at approximately the length of a mean line and that additional length may not add to accuracy any more than unlimited reobservation of a measurement will increase the accuracy of the measurement indefinitely. (It will increase only the accuracy of the value of the probable error.)

Expressed differently, the hypothesis is offered that, within a traverse, errors combine as compensating errors to a point (probably determined by the instrumentation and procedure) and then become fixed as a systematic error, increasing with the length of the traverse, and thereby establishing an accuracy, expressed in terms of error per traverse length, fixed at the point where compensation ceases.

Professors Earnest, Kissam, and Rayner in various manners raise the problem of the difference between loop closures, which close upon themselves, and position closures between fixed stations previously established by precise control.

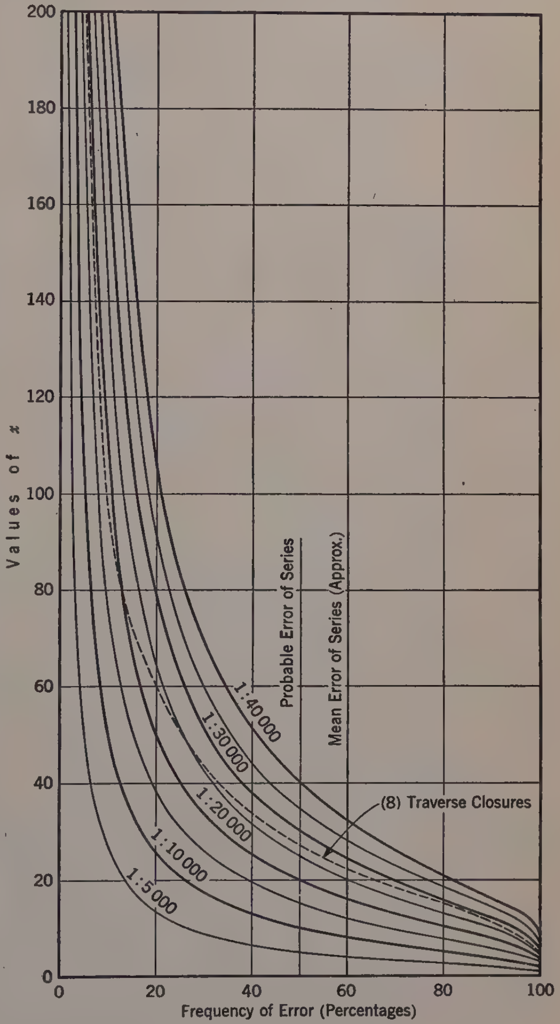

FIG. 6.—PROBABILITY CURVES (EXPRESSED AS ONE PART IN x THOUSANDS)

Although this paper is primarily concerned with actual accuracies that have been obtained in the regular business of a practicing surveyor in a territory as yet not covered by a precise control, it should not be improper (beyond the bounds of possibility) to project the same reasoning into channels indicated by the facts disclosed.

Therefore, if (as seems to be the case) traverse closures are in general equal in value to the square root of the summation of the squares of the mean linear and angular errors of the instrumentation and procedure used, the relative values of loop closure and position closure should be indicated by the use of curves 5 and 6, respectively, for E_1 in the formula $E_t = \sqrt{E^2_a + E^2_1}$ resulting in a calculated mean value of $E_t = 1 : 21{,}500$ for loop closures (an observed mean of 1 : 22,600) and a calculated mean value of 1 : 18,000 for position closure.

The latter value is considerably higher than that generally recognized as obtainable with the instrumentation, and no conclusive proof of the hypothesis can be offered, since the precise control in progress under the direction of Professor Earnest has not progressed sufficiently to permit such proof. The statement can be made that such few position closures between established transfer points, as have been run by the writer, have been better than 1 : 20,000. These runs have been too few to date to eliminate entirely the possibility of mere accident and the phenomenon of probable frequency of error.

Fig. 6 indicates theoretical probability curves for accuracies of from 1 : 5,000 to 1 : 40,000 and shows the probable error or "accuracy" of each series as well as the approximate mean error of the series, which occurs at about the 60% frequency in actual series run.

In accordance with probability formulas, for any prescribed accuracy established by instrumentation and procedure, 27% of the results of a large group of observations will show an accuracy of double (or an error of one half) that of the series, whereas 18% of the results will show an accuracy of one half (or an error of double) that of the series.

Fig. 6 also shows traverse curve 8, indicating close conformity of the actual traverse curve with the theoretical probability curve for an accuracy of about 1 : 27,000. Contrary to supposition, the frequency of extremely small errors is shown to be actually less than the theoretical.

The writer is unable to agree with Professor Kissam that "to obtain this accuracy [1 : 35,000] the average probable error would have to be 1 : 73,500." This theory is contrary to that of Professor Rayner as elaborated in his paper previously mentioned[5] and is not indicated by the results of these tabulations. Professor Kissam, however, supports this writer against Professor Rayner's criticism on the subject of the value of retaping a line. The difference of opinion on this point seems to be only in definition of terms and not in fact. The writer thanks the discussers of this paper for new thought to apply to further study of the problem of surveying accuracies.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ENTRAINMENT OF AIR IN FLOWING WATER A SYMPOSIUM

Discussion

BY MESSRS. J. H. DOUMA, AND JOE W. JOHNSON

J. H. DOUMA,[34] JUN. AM. SOC. C. E.[34a]—The past half century has witnessed the construction of a number of major hydraulic structures in which the problem of air entrainment is important. In predicting the hydraulic flow characteristics of these large structures, engineers have relied upon engineering judgment, upon empirical formulas, and, in the past decade, upon hydraulic models. Engineering judgment has been developed largely through the tedious process of trial and error. Empirical formulas have been based on experimental measurements with comparatively low velocities and no entrained air. Model experiments of hydraulic structures have not involved air entrainment. Thus, although design methods with respect to air entrainment are still irrational, the study of air entrainment in flowing water gives promise of effecting the next major advance in the design of hydraulic structures.

The results of studies on open channels, as outlined in Mr. Hall's paper, are conclusive in showing that depths and velocities in steep chutes and spillway channels cannot be calculated by the usual application of Manning's formula without consideration of entrained air. Another conclusion is that the hydraulic design of channel walls, horizontal curves, vertical curves, superelevated inverts, and stilling basins should be based on new design assumptions and formulas involving entrained air.

The Manning formula

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (50)$$

was intended for use with uniform flow in natural streams and improved channels with small slopes and low velocities. Under these conditions there is undoubtedly very little resistance to flow from the water-air surface. How-

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NOTE.—This Symposium was published in September, 1942, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: January, 1943, by Warren DeLapp, Jun. Am. Soc. C. E.; February, 1943, by Karl R. Kennison, M. Am. Soc. C. E.; and March, 1943, by Messrs. Robert T. Knapp, and Carl E. Kindsvater.

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[34a] Received by the Secretary February 8, 1943.

ever, for high-velocity flow, large volumes of air are set in motion with additional losses which are not included in the Manning formula. Thus, application of the formula to high-velocity flow requires some modification to include losses due to air entrainment. Discussion of the variables in Eq. 50 will clarify the development of a suggested modified formula.

In the Manning formula, the velocity varies as the square root of the hydraulic gradient. For uniform flow on any slope, the hydraulic gradient will be parallel to the channel invert, irrespective of air content. Thus, entrained air has no effect on the hydraulic gradient of uniform flow. For nonuniform flow, however, the hydraulic gradient is not parallel to the channel invert but is some function of depth, velocity, and air content for any given channel and discharge.

The velocity varies as the two-thirds root of the hydraulic radius. Then, according to the formula, an increase in air content and a corresponding larger hydraulic radius would result in a higher velocity. This result is quite the opposite from that experienced with aerated flow.

Values of Manning's roughness factor n have been determined for many channels with various kinds of boundary materials, but with low-velocity flows containing no entrained air. Under these conditions, the value of n is a measure of the channel-boundary roughness, as defined by resistance to the flowing water. Entirely new sets of n-values, which would include air resistance and other losses due to entrained air, could be determined for aerated flow. This procedure, however, would result in a variation in the n-value, depending on the air content, for any one channel roughness. It does not follow that a concrete channel lining, for instance, placed under similar conditions on a steep slope, should have a roughness factor different from that placed on a flat slope.

The expression $\frac{1.486}{n}$ in the Manning formula was designed to make values of n correspond to the values of Kutter's n. For aerated flow the expression can be changed to $\frac{N}{n}$, in which values of n are the same as those established for nonaerated flow and N is a function of air content. By this procedure, the original definition of n (measure of channel roughness) is retained, and the effect of air entrainment on the expression is given by the variable N.

Of the several methods of modifying Manning's formula for application to aerated flow, Mr. Hall has selected, as the most logical, one in which computed areas and hydraulic radii (assuming no entrained air) are used instead of actual areas and hydraulic radii for the aerated flow. This necessitates the use of hypothetical values of n (n_c, computed by Eq. 26) smaller than those normally used for nonaerated flow, as opposed to results obtained for aerated flow when observed hydraulic radii are used in the Manning formula.

It appears that the most logical procedure for aerated flow would be to compute actual velocities by using actual hydraulic radii and hydraulic gradients, values of n for nonaerated flow, and a variable N instead of 1.486

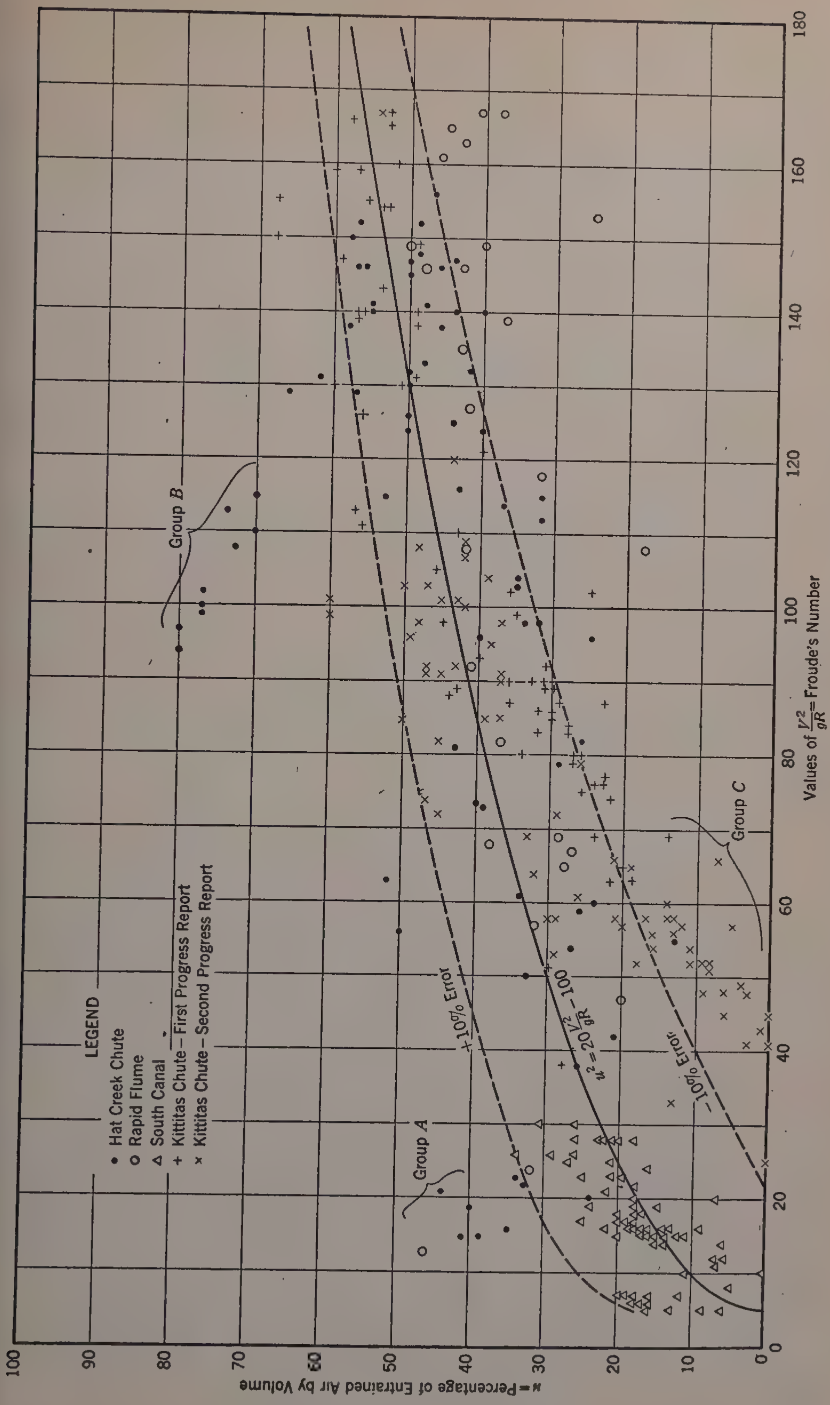

FIG. 23.—RELATION OF PERCENTAGE OF ENTRAINED AIR TO FROUDE'S NUMBER

in Manning's formula. The modified form of the formula then becomes

$$V = \frac{N}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \dots\dots\dots (51)$$

Using Eq. 51, the basic expression, Eq. 17, for flow in steep rectangular chutes becomes

$$\frac{dl}{dV} = \frac{\frac{\alpha V}{g} - \frac{q \cos \theta}{V^2}}{\sin \theta - \frac{n^2 V^2}{N^2 R^{4/3}}} \dots\dots\dots (52)$$

In Eq. 52, R and N are functions of velocity and air content. The equation can be solved by determining expressions for R and N in terms of V and known quantities.

In developing the expression for R, the data reported in the paper were used to plot percentage of entrained air, u, against $\frac{V^2}{g R}$, shown in Fig. 23. This plotting differs from Fig. 10 by the use of u instead of the more complicated ratio $\frac{1 - \rho}{\rho}$ and by the use of the hydraulic radius of the aerated flow instead of the computed hydraulic radius of water alone. Mr. Hall erroneously uses $\frac{V}{g R_c}$ as the Froude number. The Froude number is normally defined as $\frac{V}{\sqrt{g D}}$, which probably should become $\frac{V}{\sqrt{g \rho D}}$ for aerated flow.

The average of all points plotted in Fig. 23 was found to be given approximately by the relation

$$u^2 = 20 \frac{V^2}{g R} - 100 \dots\dots\dots (53)$$

The average curve was drawn to give no air entrainment for values of $\frac{V^2}{g R}$ equal to 5 or less. The expression for critical velocity at which air entrainment begins in accelerated high-velocity flow then becomes

$$V = \sqrt{5 g R} \dots\dots\dots (54)$$

Thus, for a shallow depth in any channel and a value of R equal to 1.0, for example, air entrainment would begin when the velocity is approximately 13 ft per sec. For a greater depth and a value of R equal to 5.0, air entrainment would begin when the velocity is approximately 28 ft per sec. These velocities, in general, are consistent with the meager data available on critical velocity for air entrainment. Additional observations, however, may indicate necessary modifications of Eq. 54.

In Fig. 10, Mr. Hall has drawn all but one of the curves through the origin of coordinates. This procedure results in the computation of some air entrainment for very low velocities. All of the curves should probably be drawn with a finite value of $\frac{V^2}{g\,R_c}$, determined by the critical velocity for air entrainment, when the ratio $\frac{1-\rho}{\rho}$ is zero.

On first inspection, the wide scattering of points in Fig. 23 would seem to cast serious doubt on the application of a single average curve to the several structures. However, by plotting + 10%-error and − 10%-error curves practically all of the plotted points fall within these limits of error. The curves were computed by assuming errors of ± 10% in both the observed depths and velocities of the aerated flow. In view of the difficulties in obtaining accurate depth and velocity measurements, errors of 10% certainly are not excessive.

Because of turbulent entrance conditions the points in group A, Fig. 23, show high percentages of air. In group B, the high percentages of air are caused by flow leaving the chute at a sharp vertical curve. The low air content of the group C points appears to be due to errors in determination of observed velocities.

The observed data listed in Table 9(*a*) were taken from the Bureau of Reclamation's second progress report (unpublished) on studies of the flow of water in open channels with high gradients. In this report the average observed velocity was determined from the oscillograms by taking the time of travel between two sets of electrodes, as given by the distance between centers of gravity of areas under two successive curves representing the passage of a salt cloud past the upper and lower stations. In the first progress report (unpublished) the time distance was obtained by averaging the distances between beginning, maximum, and other characteristic points on the curves. Velocities determined in the first progress report average 24% higher than velocities determined in the second progress report.

The data for Kittitas chute from both progress reports are plotted in Fig. 23. The data from the first progress report, in general, are consistent with the data for the other three structures. In the lower velocity range (group C region), data from the second progress report are not in agreement. The data in Table 9(*a*) show the critical velocity for air entrainment in the Kittitas chute to be approximately 40 ft per sec. Photographs of flow in the upper end of the chute, however, indicate considerable entrained air at stations where velocities of about 40 ft per sec occurred. Thus, the group C points appear to be in error, and the velocities and percentage of air reported in Table 9(*a*) are questionable.

Returning to the solution for R, Eq. 53 can be rewritten to give

$$R = \frac{20\,V^2}{g(u^2 + 100)} \quad \text{......................} \quad (55)$$

In Eq. 55, u can be eliminated as follows: The basic aerated flow formula, as given by Eq. 3, is $Q = \rho\,A\,V$. In this equation,

$$\rho = 1 - \frac{u}{100} \quad \text{......................} \quad (56a)$$

and

$$A = \left(b + \frac{2\,Q}{\rho\,b\,V}\right) R \quad \text{..................} \quad (56b)$$

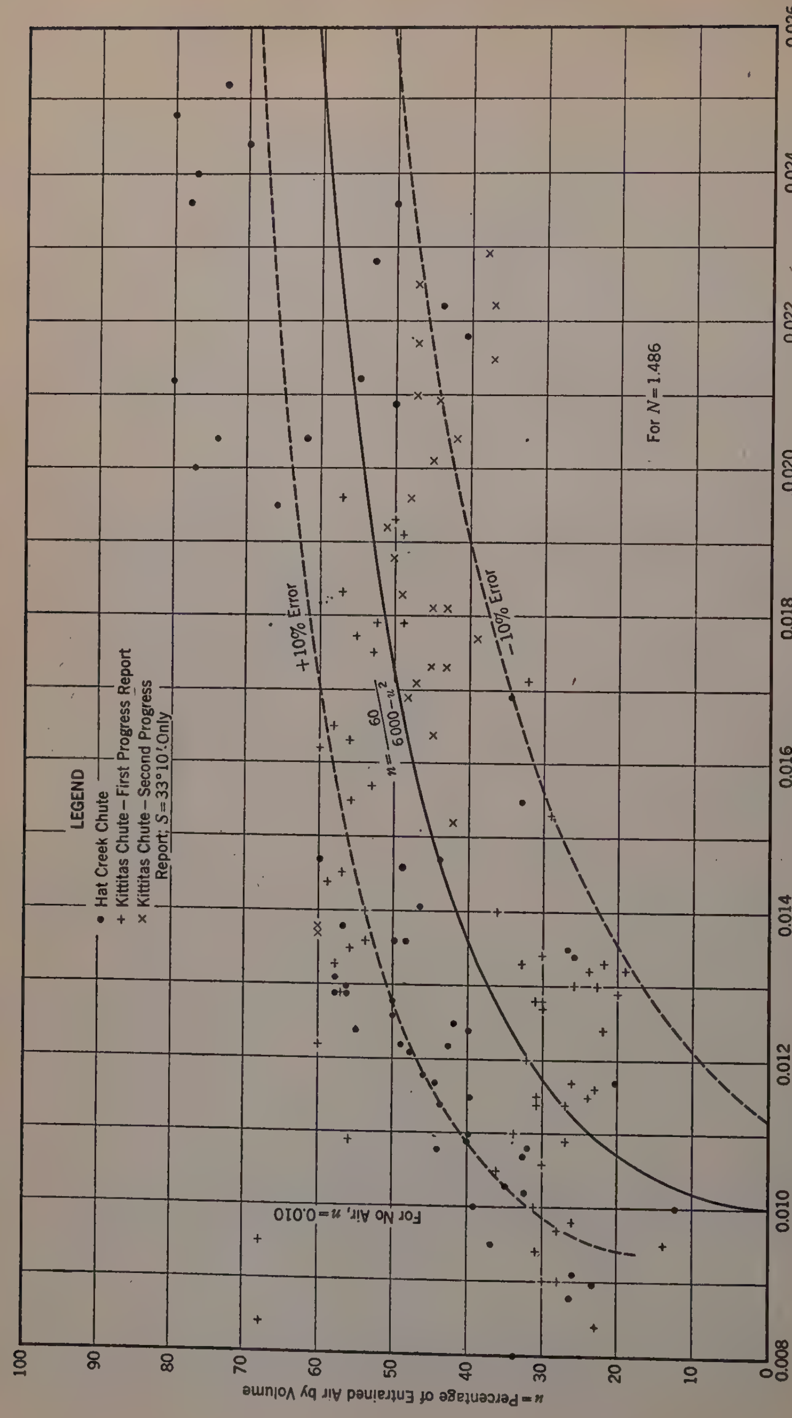

FIG. 24.—RELATION OF PERCENTAGE OF ENTRAINED AIR TO MANNING'S *n*

Solving Eqs. 3 and 56 simultaneously:

$$u = 100 + \frac{200\,Q}{b^2\,V} - \frac{100\,Q}{b\,R\,V} \quad \text{(57)}$$

Finally, by substitution of Eq. 57 in Eq. 55, the following quadratic expression for R in terms of actual velocity and known quantities is obtained:

$$g\,(1.01\,b^4\,V^2 + 4\,b^2\,V\,Q + 4\,Q^2)\,R^2 - (0.002\,b^4\,V^4 + 2\,g\,b^3\,V\,Q + 4\,g\,b\,Q^2)\,R + g\,b^2\,Q^2 = 0 \quad \text{(58)}$$

Using the binomial theorem, values of R in Eq. 58 can be determined for any given channel and design discharge and for several values of V. A curve of R against V may then be plotted to facilitate the water surface computations.

In developing an expression for obtaining values of N in Eq. 52, data for Hat Creek and Kittitas chutes were plotted in Fig. 24 to show the variation of Manning's n with air content when an N-value of 1.486 is used for aerated flow. Mr. Hall states that values of n obtained from observed areas and velocities of aerated flow are equivalent to those observed in normal channels (nonaerated flow). This implies that the value of n does not vary with air content. Fig. 24, however, indicates a definite increase in n with an increase in u. The average relationship is given by

$$n = \frac{60}{6{,}000 - u^2} \quad \text{(59)}$$

Fig. 24 indicates (although not conclusively) that the value of n would be 0.010 for the concrete chutes if the flows contained no entrained air. This n-value is applicable to smooth concrete chutes. From the description of the Hat Creek and Kittitas chute linings an n-value of about 0.014 might be estimated. The explanation of the low n-value for relatively rough concrete lining is not known.

Using a constant value of n equal to 0.010, the data used in plotting Fig. 24 were recomputed for values of N. The results are plotted in Fig. 25. The average relationship of u and N is given by:

$$N = 1.486 - 0.000248\,u^2 \quad \text{(60)}$$

Eq. 60 can be used to obtain values of N in the solution of Eq. 52. The maximum value of N is 1.486, when the flow contains no entrained air, and N decreases with increase in air content.

To check the use of Eqs. 52, 53, 58, and 60 for aerated flow, hydraulic properties computed by these equations for 395 cu ft per sec in Hat Creek chute are compared with observed values in Table 13. The comparison shows good agreement except for the stations at the sharp vertical curve.

With reference to the design of vertical curves to fit the trajectory of high-velocity flow, the method described by Mr. Hall requires the coordinate system to be parallel and normal to the slope of the chute upstream from the vertical curve. Experience has proved that computations of stations and elevations on vertical curves are simplified by using a vertical and horizontal coordinate system.

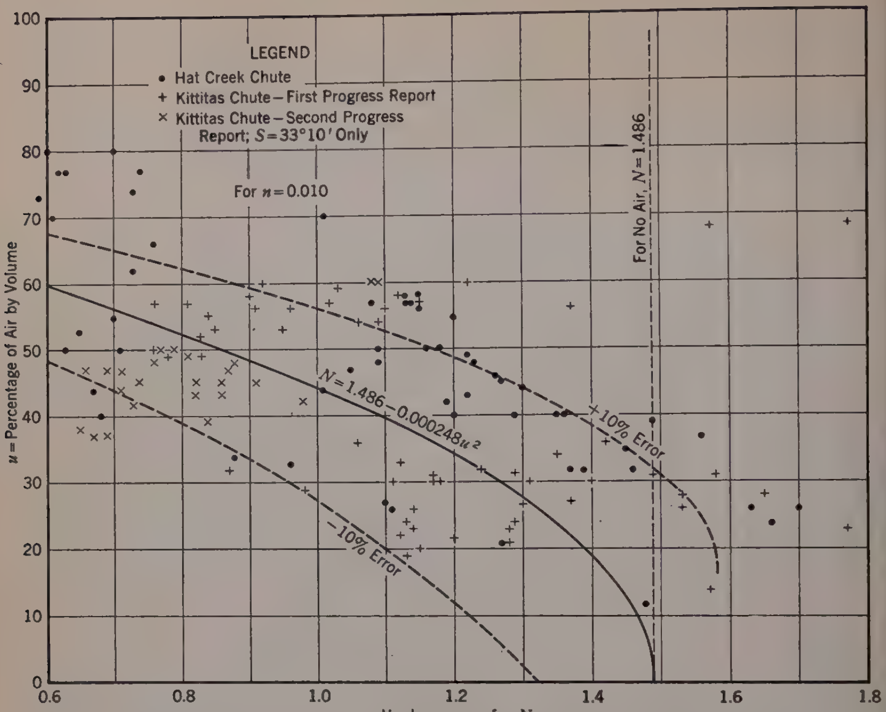

FIG. 25.—RELATION OF PERCENTAGE OF ENTRAINED AIR TO VALUES OF N

TABLE 13.—HYDRAULIC PROPERTIES, HAT CREEK CHUTE
(Q = 395 CU FT PER SEC)

Station	DEPTH (FT)		HYDRAULIC RADIUS (FT)		VELOCITY (FT PER SEC)		ENTRAINED AIR (%)	
	Observed y_o	Computed y	Observed R_o	Computed R	Observed V_o	Computed V	Observed u_o	Computed u
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0+00	4.00	....	1.74	....	28.5	....	38.5	....
0+25	3.15	3.15	1.57	1.57	33.4	33.4	33.0	33.0
0+50	2.35	2.30	1.36	1.33	40.8	40.2	26.0	25.6
0+75	2.00	2.17	1.23	1.27	48.1	45.9	25.9	30.8
1+00	1.85	2.09	1.18	1.23	54.7	50.0	29.0	34.1
1+25	1.80	2.04	1.16	1.20	60.5	53.3	34.2	36.9
1+50	1.77	2.01	1.15	1.19	65.0	56.0	37.4	39.0
1+75	1.85	2.00	1.18	1.19	68.0	58.2	43.6	41.0
2+00	1.80	1.99	1.16	1.18	71.7	60.0	44.5	42.1
2+25	1.80	1.98	1.16	1.18	73.9	61.0	46.2	43.0
2+50	1.90	1.97	1.20	1.18	75.3	61.7	50.0	43.5
2+75	1.85	1.97	1.18	1.18	75.9	62.0	49.0	43.7
3+00	3.52	1.96	1.59	1.18	76.0	62.3	73.8	44.2
3+25	4.60	1.95	1.83	1.18	75.8	63.9	80.0	45.6
3+50	3.50	1.95	1.64	1.18	74.9	65.2	73.2	46.2
3+75	1.97	1.96	1.23	1.18	72.3	64.2	49.8	45.6
4+00	1.95	1.97	1.21	1.18	61.1	61.8	39.9	43.7
4+09.5	2.17	1.98	1.30	1.18	57.9	60.3	43.4	42.6

Referring to Fig. 26, the equation of the vertical curve is derived as follows: With the origin of coordinates at the beginning of curvature of the vertical curve, where the maximum computed velocity for aerated or nonaerated flow is V_o, the horizontal distance covered by the flow in time t is given by

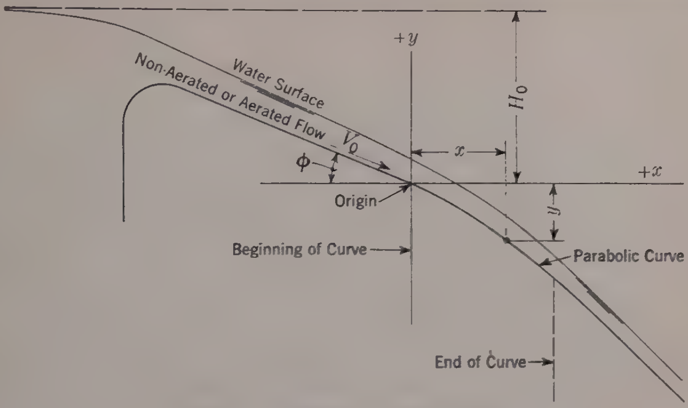

FIG. 26.—VERTICAL INVERT CURVE

$$x = t\, V_o \cos\theta \qquad (61a)$$

and in this same time the flow would drop a vertical distance

$$y = -\left(t\, V_o \sin\theta + \tfrac{1}{2}\, g\, t^2\right) \qquad (61b)$$

Eliminating t in Eqs. 61*a* and 61*b*, the equation for the curve is

$$y = -\left(x \tan\theta + \frac{g\, x^2}{2\, V_o^2 \cos^2\theta}\right) \qquad (61c)$$

As suggested by Mr. Hall, V_o, in Eq. 61*c*, should be taken as approximately 1.2 times the computed mean velocity at the beginning of the curve. For short chutes V_o may be determined from $\sqrt{2\, g\, H_o}$, in which H_o is the total vertical drop. Eq. 61*c* then becomes

$$y = -\left(x \tan\theta + \frac{x^2}{4\, H_o \cos^2\theta}\right) \qquad (61d)$$

Eq. 32 for superelevation in a horizontal curve is applicable to tranquil flow but not to rapid flow (supercritical velocity). For rapid flow, the maximum superelevation is considerably greater than given by Eq. 32; and it is not uniform around the bend but has maximum and minimum zones which result from transverse wave fronts. In addition to the rise in water surface at the outside wall caused by centrifugal action, there is a rise or drop in water surface caused by the attacking or receding wave front.[35] The maximum superelevation along the outside wall is given by

$$e_s = \frac{1.2\, V^2\, b}{g\, r_c} \qquad (62)$$

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[35] "Testing Theoretical Losses in Open Channel Flow," by J. G. Jobes and J. H. Douma, *Civil Engineering*, November, 1942, pp. 613–615.

Observed data listed in Table 11 have been retabulated in Table 14 for comparison with the difference in water surface at the inside and outside walls computed by Eqs. 32 and 62. Computed differences in the last line of the table are maximum values that would occur if the maximum height of transverse wave occurred at each of the stations. The observed values in all cases are smaller than the maximum values. The maximum height of transverse wave probably did not occur exactly at any of the stations listed. The minimum height occurred near some stations. The initial transverse wave at the point of curvature of the 142-ft curve developed by upstream curves is another factor that caused the maximum observed superelevation to be smaller than computed. Transverse waves at the beginning of a curve may either magnify or dampen the new disturbance caused by the curve. From the data it appears that the disturbance was dampened in the 142-ft curve.

TABLE 14.—COMPARISON OF OBSERVED AND COMPUTED SUPERELEVATION IN SOUTH CANAL CHUTE

(Radius of Curve, 142 Ft)

Description	23+00	23+25	23+50	23+75	24+00	24+25	24+50	24+75	25+00	25+25	25+50
Observed velocity, in feet per second	16.6	19.2	21.7	25.2	28.8	29.6	30.5	32.0	33.5	34.3	35.1
Difference in Water Surface at Outside and Inside Walls, in Feet:											
Observed......	0.87	0.25	1.30	0.90	1.20	1.15	1.55	1.30	2.75	2.65	2.00
Computed by—											
Eq. 32.......	0.60	0.69	0.72	0.97	1.27	1.34	1.43	1.57	1.71	1.80	1.88
Eq. 62.......	1.44	1.64	1.71	2.31	3.04	3.20	3.42	3.76	4.11	4.31	4.52

In view of the complex wave pattern that may be developed when a series of closely spaced horizontal curves is located in a chute, the water surface cannot be computed accurately, and, for the sake of efficiency and economy, model studies should be made prior to construction. When a chute has but one horizontal curve, or horizontal curves, spaced some distance apart, the maximum superelevation can be computed by use of Eq. 62. The usual design practice is to provide ample freeboard for the maximum superelevation over the entire length of the curve and for some distance in the downstream tangent although the maximum superelevation will occur at only a few stations. Model studies of structures so designed would probably effect some saving by reduction of wall heights. In some designs superelevating the invert reduces costs. Superelevation in accordance with Eq. 32 will not eliminate the development of transverse waves, but properly designed compound curves, which produce disturbance waves of the same amplitude but of opposite phase from those produced by the main curve, will reduce their height.

The design of hydraulic-jump stilling basins is another problem affected by air entrainment. The commonly used formula for jump height in a rectangular channel is

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d^2_1}{4} + \frac{2\, d_1\, V^2_1}{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

in which d_1 and V_1 are the depth and velocity upstream of the jump and d_2 is the depth after the jump. Although Eq. 63 is not applicable to aerated flow, a formula for aerated flow can be derived by the same method.

Referring to Fig. 27, the formation of a stable hydraulic jump must be accompanied by a change in kinetic momentum from section (1) to section (2) equal to the difference in forces due to hydrostatic pressures at sections (1) and (2). The change in kinetic momentum per second with mass flow, $\frac{q\,w}{g}$, per foot of width in a rectangular channel is

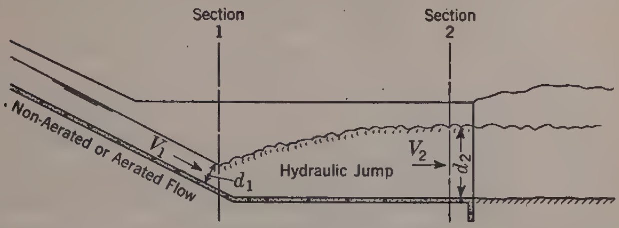

FIG. 27.—STILLING BASIN

$$M_2 - M_1 = \frac{q\,w}{g}(V_2 - V_1) \quad \text{..................} (64a)$$

The resultant force between sections (1) and (2), assuming no entrained air at section (2), is

$$F_1 - F_2 = \frac{w}{2}(\rho\,d^2_1 - d^2_2) \quad \text{..................} (64b)$$

A third equation is

$$q = \rho\,d_1\,V_1 = d_2\,V_2 \quad \text{..................} (64c)$$

Equating Eqs. 64*a* and 64*b* and eliminating V_2 by using Eq. 64*c*, the formula for jump height with aerated flow becomes

$$d^2_2 = \rho\,d^2_1 + \frac{2\,\rho\,d_1\,V^2_1}{g}\left(1 - \rho\frac{d_1}{d_2}\right) \quad \text{..............} (64d)$$

Eq. 64*d* can be solved for d_2 by trial and error.

The following sample computations are made to illustrate the influence of entrained air on the design of stilling basins: Assume a design discharge of 200 cu ft per sec per ft for a wide spillway channel. Further, assume $V_1 = 100$ ft per sec as calculated by the usual method for nonaerated flow. Then $d_1 = 2.0$ ft and solution of Eq. 63 gives $d_2 = 34.2$ ft. Now, for purposes of comparison, assume $V_1 = 80$ ft per sec as computed by Eq. 52 for aerated flow. Fig. 23 gives an air content of 31% from which $\rho = 0.69$. From Eq. 64*c*, $d_1 = 3.6$ ft; and from Eq. 64*d*, $d_2 = 30.2$ ft, which is 4.0 ft less than computed by the usual method.

Frictional resistance due to entrained air reduces the mean velocity and total kinetic energy at section (1), requiring a smaller d_2 depth for the formation of the hydraulic jump. The result is a significant saving in stilling-basin wall heights, reduced floor thickness because of smaller uplift pressures, less excavation, and a shorter stilling-basin length.

Many additional experimental data and studies by a number of hydraulic engineers are needed to confirm the application of the aerated flow formulas to steep chutes and spillways. In view of usual conservative assumptions of the value of n, for purposes of design, and the use of liberal freeboard, further studies should probably be directed toward establishing approximate general formulas before much effort is given to complex refinements.

JOE W. JOHNSON,[36] ASSOC. M. AM. SOC. C. E.[36a]—Valuable quantitative data have been supplied, by this Symposium, to a field which previously has been discussed only in rather broad generalities. The various data should prove important in future design problems where air entrainment is a factor.

Of interest to the writer was the last part of the paper by Messrs. Kalinske and Robertson. In Fig. 16 experimental data are shown to verify the momentum relationship for a hydraulic jump in a 6-in. pipe. The authors attribute the scatter of the points around a 45° line to the difficulty in measuring values like depth of flow and pressure. The fact, however, that most of the points fall consistently below the 45° line indicates that experimental errors are not entirely the cause of scatter but suggests instead that some other factor may have been in error or was neglected in developing the equation.

A possible explanation of the consistent shift of the points to a position below the 45° line is the fact that the air-water flow ratio, β, may not have had the same value as the air-water ratio of the mixture (hereafter termed Δ). This later ratio refers to the mixture as actually existing between two points in the pipe immediately below the jump where the pipe is completely filled with the flowing mixture. The two ratios β and Δ have equal values only when the air and water are moving at the same velocity. When the air and water are not moving at the same velocity, a "slip" loss occurs, in which case the term β in Eqs. 37 and 38 should be replaced by Δ, the ratio of air to water as actually existing in the pipe.

A convincing demonstration of how great may be the difference between these two ratios can be obtained from published data on the flow of gas-liquid mixtures through vertical pipes.[37] In these experiments tests were made on the flow of water, kerosene, and oil through pipes, 1, 2, $2\frac{1}{2}$, 3, and 4 in. in diameter. The flow ratio β was determined by metering the inflows. The volume ratio Δ was obtained by trapping a volume of the flowing mixture between quick-acting gate valves located at two points in the vertical pipe. Then by draining and determining the volume of the trapped liquid and with the volume of the pipe between the gate valves being known, the ratio of the mixture Δ was computed.

A typical example of the observations taken during these experiments is given by run 1 of the tabulated data. Thus, by metering the inflow to the apparatus, 15.7 cu ft per min of gas at mean pipe pressure and 0.86 cu ft per min of water were flowing in the pipe, thereby giving a gas-water flow ratio β of 18.3. The gas-water ratio of the mixture Δ obtained by closing the quick-acting gate valves was found to be only 3.2. The linear velocity of the water was computed to be 10.5 ft per sec, and the velocity of the gas was 56.1 ft per sec.

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[36] Asst. Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

[36a] Received by the Secretary March 1, 1943.

[37] "Experimental Measurement of Slippage in Flow Through Vertical Pipes," by T. V. Moore and H. D. Wilde, *Transactions*, A. I. M. E., Petroleum Div., 1931, pp. 296–313.

The wide difference that is possible between the two values of the gas-water ratios is the basis of the foregoing statement that the value of β in Eqs. 37 and 38 perhaps should be replaced by the ratio Δ. In attempting to compare the experiments of T. V. Moore and H. D. Wilde[37] with those of Messrs. Kalinske and Robertson, a comment on the flow conditions in the two experiments is necessary. In the Moore and Wilde experiments the gas expanded as it flowed upward through the pipe and "slipped" past the water. The opposite condition probably occurred in the Kalinske-Robertson experiments—namely, the water was "dragging" air along from a low pressure space to a high pressure space and the velocity of the water slightly exceeded the velocity of the air. Because of this velocity difference, therefore, the mixture ratio Δ was greater than the flow ratio β. Thus, if β in Eqs. 37 and 38 is replaced by Δ, the value of M_2 would be increased and the value of W would be decreased. The net result increases the ordinate of the various experimental points shown in Fig. 16. Closer grouping around the 45° line probably would result. The installation of two quick-closing gate valves in the Kalinske-Robertson apparatus should give interesting information on the relative magnitude of β and Δ. Obviously, their values would not differ as greatly as in the experiments of Messrs. Moore and Wilde where the gas was expanding.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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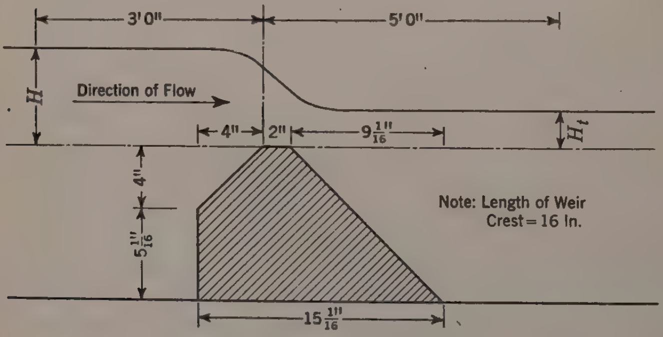
DISCUSSIONS

CONFORMITY BETWEEN MODEL AND PROTOTYPE
A SYMPOSIUM

Discussion

By Messrs. Graham Walton, H. A. Einstein, K. G. Tower, R. J. Pafford, Jr., Edward H. Schulz, and F. T. Mavis

Graham Walton,[20] Assoc. M. Am. Soc. C. E.[20a]—The information on submerged weir discharge in Mr. Soucek's paper is of particular interest. Weirs are frequently installed in previously constructed channels to measure sewage or industrial wastes. Restrictions in available head loss may require submerged flow conditions, at least during periods of peak flow.



Note: Length of Weir Crest = 16 In.

Fig. 70.—Weir Cross Section

In 1936 a weir of trapezoidal cross section (Fig. 70), with a surface of smooth cement-sand mortar, was calibrated at the hydraulic laboratory of the University of Wisconsin, at Madison, for the express purpose of measuring waste discharge at a paper mill. Special attention was given to the investigation of

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Note.—This Symposium was published in October, 1942, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1942, by A. E. Niederhoff, Assoc. M. Am. Soc. C. E.; January, 1943, by Messrs. C. I. Grimm, and Joe W. Johnson; February, 1943, by V. L. Streeter, Assoc. M. Am. Soc. C. E.; and March, 1943, by Glen N. Cox, M. Am. Soc. C. E.

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[20a] Received by the Secretary February 19, 1943.

the effect of submergence on the discharge. The heads were measured, 3 and 5 ft, respectively, upstream and downstream from the upstream edge of the weir crest.

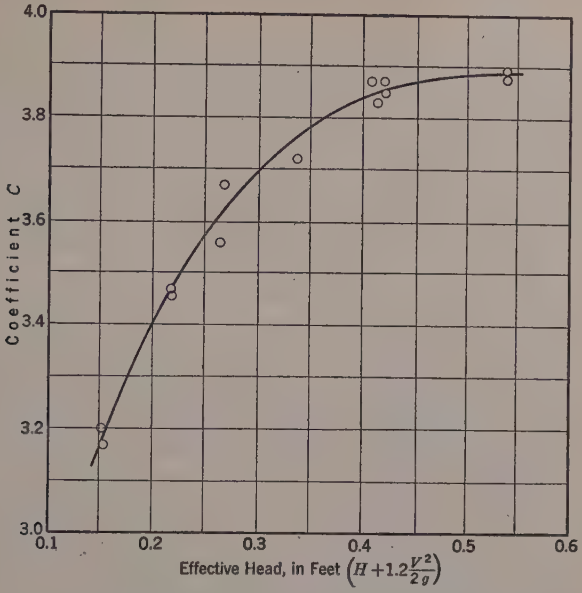

FIG. 71.—COEFFICIENT *C* FOR WEIR WITH FREE DISCHARGE

Test results are summarized in Fig. 71, which gives the coefficient of discharge *C*, as determined for

$$Q = C\,L\left(H + 1.2\,\frac{V^2}{2\,g}\right)^{\frac{3}{2}} \qquad (24)$$

Fig. 72 shows the ratio of submerged to free discharge for various submergences. The latter is plotted as a single-valued function and is surprising in its agreement with U. S. deep waterways tests (42*a*).[20b] There is some indication, however, that, had it been possible to increase the distance at which the head downstream was measured, a family of curves might have been obtained. Careful examination of the original data shows abnormal change in downstream head with transition from plunging to flowing nappe. This should not be the case with constant discharge if the control section downstream from the weir alone determined the elevation of water surface.

A comparison of Fig. 72 with information shown in Figs. 52 and 53 emphasizes the fact that the effect of submergence on discharge is determined to a considerable extent by the cross section of the weir.

The variation of the upstream head on the weir due to the change from plunging to flowing nappe is noted in Table 11. For the same discharge, over this particular trapezoidal weir, the head for flowing nappe averages 2.5%

[20b] Numerals in parentheses, thus: (42*a*), refer to corresponding items in the Bibliography, which appears as the last unit of the Symposium, and at the end of discussion in this issue.

greater than that for plunging nappe. This indicates a corresponding decrease in the coefficient C of 3.7%, or the error which may result because of failure to note type of flow. The possible effect of plunging and flowing nappe still should be considered.

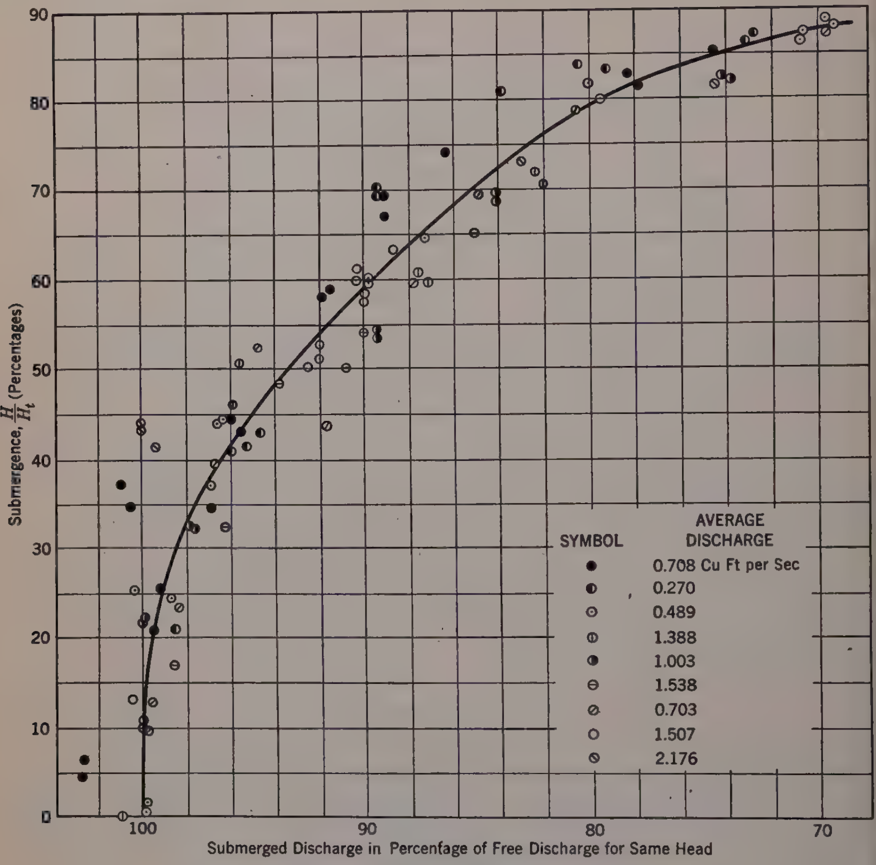

Fig. 72

TABLE 11.—Variation of Head on a Weir with the Transition from a Plunging to a Flowing Nappe

Description	Discharge, in Cubic Feet per Second:								
	0.268	0.490	0.704	0.707	0.999	1.389	1.509	1.536	2.16
Discharge for flowing nappe conditions (cu ft per sec)			0.702	....	....	....	1.502	1.524	2.14
Head on Weir (Ft):									
Plunging nappe	0.153	0.220	0.266	0.264	0.329	0.413	0.425	0.443	0.538
Flowing nappe	0.156	0.228	0.273	0.274	0.346	0.424	0.432	0.442	0.546
Weir Submergence (%) for:									
Plunging nappe	21.2	44.3	46.1	36.0	32.4	48.4	52.0	48.2	59.6
Flowing nappe	42.2	60.7	48.0	43.8	54.2	60.1	59.6	52.7	63.8
Transition (±)	36	39?	42?	....	47	51	53	....	60

Although recognizing the value of measuring the downstream head at a distance beyond the influence of nappe conditions, the writer would like to discuss the limitations encountered in practice. This head measurement must be made at a location which is not affected by nappe conditions or by the down-drop curve resulting from the downstream control section. Also, it may be necessary to correct for the drop in hydraulic gradient utilized in overcoming resistance to flow. Frequently conditions that necessitate use of submerged weirs for flow measurement limit the available distance between which the two heads may be measured. For many submerged weirs used as measuring devices, the head downstream must be determined within that region affected by nappe conditions, particularly when flowing nappe discharge occurs. Thus, it is recommended that such weirs be designed, in so far as possible, to prevent flowing nappe conditions.

H. A. EINSTEIN,[21] ASSOC. M. AM. SOC. C. E.[21a]—The different papers of this Symposium describe a great number of hydraulic models and their prototypes, compare the behavior of both, and generally prove that a carefully devised model is an extremely reliable instrument. The engineer in the laboratory who builds and operates models will appreciate especially these comparisons because, more than anybody else, he knows the possible reasons why similitude may be disturbed.

In several of the papers, friction has been given as the cause for deviation between model and prototype. It is known that friction along rough walls will conform, in different scales, according to Froude's law if the roughness is reduced in the model according to the scale of all other lengths and if this reduced roughness combined with the reduced Reynold's number of the flow still represents a rough wall. If this is not the case, disagreement is inevitable. This case occurs most frequently in connection with hydraulic structures, which are generally rather smooth in the prototype to reduce friction. Usually, this friction is unimportant or may be corrected by calculation. There are cases, however, in which the true reproduction of this friction is essential. In these cases a method of distortion may be used which proves very effective for river models, many of which must be distorted to keep the flow turbulent. While visiting the hydraulic laboratory in Vicksburg, Miss., where such models usually can be found, the writer was astonished to see how much the natural roughness of the model must be increased to counteract the effect of the distortion on the friction. It is clear, therefore, that a comparatively slight distortion of any hydraulic model will allow an effective increase of the wall roughness which may just make the difference between rough and smooth.

This raises the question of the reliability of distorted hydraulic models in general. It was somewhat disappointing that no quantitative comparison of any such model with its prototype could be offered in this Symposium. Elsewhere (17), the writer, with R. Müller, has described the conditions under which a distorted model with movable bed will conform with the prototype, giving the results of the application of the method to a river model.

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[21] Hydr. Engr., SCS, U. S. Dept. of Agriculture, Greenville, S. C.

[21a] Received by the Secretary February 26, 1943.

The model in this case was that of the Rhine River above the Lake of Constance (62). Within the first 20-mile stretch above the lake the stream loses the greater part of its bed load by sedimentation in its own bed. This gradual decrease of the rate of bed-load transport down this stretch is parallel with a gradual decrease of the slope. The alarming speed with which the bed was filling up forced the responsible authorities to study all possible ways of relieving the dangerous situation. The model in question was intended to represent a section of the river in the upper part of the 20 miles and to reveal constructive measures which would increase the river's capacity for bed load without increased slope. The rules for transportation in the model had to be found empirically because no theory of any kind existed at the time. Therefore, a series of trials was made to duplicate, in the model, the changes of the river bed that had taken place during past years using Froude's law for the flow. When agreement was satisfactory, the main experiments were started. However, only the preliminary experiments can be used in comparing the model with the prototype, because the corresponding measurements in the prototype applied to them only.

The measurements in the river, in addition to data not used in this connection, include (*a*) several sets of cross sections, (*b*) the hydrographs for the time intervals between these surveys, and (*c*) velocity and slope measurements at different stages. Mechanical analyses of the bed material were available for the entire 20-mile stretch. A set of bed-load measurements using a calibrated trap gave the rates of bed-load movement at one section for different stages and checked very well with calculations using the bed-load formula, commonly referred to as the Swiss formula. This same formula was used to calculate the transportation at other sections of the stretch, and the difference of transport during certain periods of time was checked against the deposition between the sections and in the lake. The measurements within this rather intricate system agreed astonishingly well, thus introducing the rates of transportation into the list of variables measured in the prototype.

The model was built entirely empirically. Originally it was supposed to be geometrically undistorted except for the grain size in the movable bed which was composed of coal. The low specific gravity of the grains should counteract the effect of the large grain sizes as far as transportation is concerned. However, it was known at the time from similar models that the time ratio chosen for transportation must be entirely different from the time ratio of the flow proper. By trial and error a ratio of 360 gave, very closely, the same configuration of the model bed as was found in the prototype and was used thereafter throughout the entire study.

All the evidence seemed to indicate that the model was similar to the prototype, but the quantitative rules of this similarity were not discovered until more than a year later, when Froude's law was abandoned as a basis of comparison and was replaced by a new system of equations describing similarity in a distorted model.

Instead of a general ratio for all different variables of the dimension of length, four different ratios were used:

(1) λ for lengths in direction of the main flow;
(2) ϕ for the width, a horizontal length perpendicular to the main flow;
(3) χ for the height, any vertical length; and
(4) δ for the grain diameters.

An additional distortion was allowed by the separate introduction of

(5) ξ for the ratio of slopes, independent from λ and χ.

Each common Greek letter represents the ratio of a certain measure in nature over the corresponding measure in the model. Also, for time, a distortion is allowed by the separate introduction of:

(6) τ for any time concerning the movement of water; and
(7) θ for any time concerning the movement of sediment.

A number of constants enter any hydraulic problem involving bed-load movement. The acceleration of gravity and the density of the water are assumed to be equal in model and prototype:

(8) η may be the ratio of the effective specific gravities of the sediments under water. Their specific pore volumes in the bed are assumed to be equal;

(9) ζ is the ratio of the constants in the Manning-Strickler formula as explained subsequently; and

(10) σ is the ratio of the constants F introduced in the bed-load formula (63).

For convenience, three more ratios of composite variables are introduced and defined by the three equations:

(11) γ as the ratio of the specific bed-load transport defined by

$$\gamma\,\theta = \eta\,\lambda\,\chi \quad \ldots\ldots (25a)$$

(12) μ as the ratio of corresponding masses of water defined by

$$\mu = \lambda\,\phi\,\chi \quad \ldots\ldots (25b)$$

and

(13) ψ as the ratio of corresponding discharges, defined by

$$\psi = \mu\,\tau^{-1} \quad \ldots\ldots (25c)$$

Four more equations can be introduced describing similarity between model and prototype. For the distorted model the general Froude's law reduces to

$$\chi = \lambda^2\,\tau^{-2} \quad \ldots\ldots (25d)$$

expressing the fact that any heights are correlated like values of $\frac{v^2}{2\,g}$. The next equation expresses the similarity of corresponding bed friction. Friction in a stream with bed-load movement is usually of the rough type, especially if the bed material is as coarse as that in the Rhine River. Therefore, friction may be described by Manning's formula. The same formula must be used in both model and prototype; otherwise the principal characteristic of similitude is lost, because corresponding values of the same type must be correlated by a constant factor. The constant n in Manning's formula can be expressed for a granular bed, according to A. Strickler (64) as the sixth root of a representative grain diameter of the bed divided by another constant. This new constant will vary only slightly for extremely low Reynolds numbers and with the intensity Φ of bed-load transportation. In ratios the Manning-Strickler formula, therefore, reads

$$\lambda\,\tau^{-1} = \zeta\,\delta^{-1/6}\,\xi^{1/2}\,\chi^{2/3} \qquad (25e)$$

The two remaining equations are derived from the postulated identity of the transportation formula. Using the (Ψ–Φ)-relationship, the values of both Ψ and Φ must be the same for corresponding points in model and prototype as long as the same (Ψ–Φ)-curve holds in both scales. If the Reynolds number of the flow in the model is extremely low (as was the case for the Rhine River model), a slightly different (Ψ–Φ)-curve applies. This difference of curve is taken care of by the introduction of ratios (14–15) α and β for corresponding Φ and Ψ values, respectively:

$$\alpha = \gamma\,\sigma^{-1}\,\eta^{-3/2}\,\delta^{-3/2} \qquad (25f)$$

and

$$\beta = \eta\,\delta\,\xi^{-1}\,\chi^{-1} \qquad (25g)$$

Flume experiments with the model sediment under flow conditions similar to those in the model resulted in a (Ψ–Φ)-curve that is lower and flatter than normal, suggesting that both α and β are smaller than unity.

Even when this method of comparison was used, the similitude did not seem satisfactory. Only when the model was interpreted as duplication of the flow at another section of the river farther downstream in the 20-mile stretch was satisfactory similitude achieved.

Table 12 shows a comparison of the values for the fifteen ratios as derived from direct measurements in model and prototype (Col. 6) with those computed according to Froude's law (Col. 3), with those using Eqs. 25 for a distorted model introducing α and β equal to 1 (Col. 4) and for a distorted model choosing α and β according to special flume studies (Col. 5). Only normal flows were studied in the model. The measured length ratio λ as well as the χ values, therefore, were obtained from the comparison of the characteristic average form of the sediment bars in model and prototype. The model appears to be almost undistorted geometrically except for being tilted downstream, as shown in Table 12.

Parameters ψ, η, and ϕ have been chosen; θ was determined by trial and error; and α, β, ζ, and σ are measured independent of the model. A comparison of the values in Cols. 5 and 6 shows remarkably close agreement of all the seven remaining calculated ratios with the measurement. Col. 4, Table 12, may illustrate how easily the entire system can be disrupted by the wrong assumption of any constants in the equations involved. These data may prove that a quantitatively similar river model with light-weight material in the bed is possible and that its behavior can be predicted. A certain distortion, especially of the time scale, is unavoidable. Because of these distortions Froude's law cannot be applied. Instead, a system of empirical formulas with their constants describing bed friction and bed movement determines the different ratios. The same equations must be applicable to both model and prototype if similarity is to prevail. All constants in these equations must be known for both scales if the different ratios are to be predicted.

TABLE 12.—COMPUTED AND MEASURED RATIOS FOR THE RHINE RIVER MODEL

Ratio	Symbol	CALCULATED			Measured
		Froude	Distorted		
(1)	(2)	(3)	(4)	(5)	(6)
Φ-values	α	1	1	0.595	0.5–0.7
Ψ-values	β	1	1	0.838	0.8–1.0
Width	ϕ	100	100	100	100
Time (sediment)	θ	10	360	360	360
Strickler-constant	ζ	1	0.80	0.80	0.80
Density (sediment)	η	1	7.7	7.7	7.7
Settling-constant	σ	1	1	1	1
Discharges	ψ	10^5	10^5	10^5	10^5
Length	$\lambda = \alpha\, \beta^{9/4}\, \phi^{-1/3}\, \theta\, \zeta^{-9/2}\, \eta^{-7/4}\, \sigma\, \psi^{1/3}$	100	276	110	100–130
Height	$\chi = \phi^{-2/3}\, \psi^{2/3}$	100	100	100	95–104
Slope	$\xi = \beta^{1/2}\, \zeta^{-3}\, \eta^{-1/2}$	1	0.704	0.645	0.6–0.65
Time (hydraulic)	$\tau = \alpha\, \beta^{9/4}\, \theta\, \zeta^{-9/2}\, \eta^{-7/4}\, \sigma$	10	27.6	11.0	9.3–13.7
Specific transportation	$\gamma = \alpha\, \beta^{9/4}\, \phi^{-1}\, \zeta^{-9/2}\, \eta^{-3/4}\, \sigma\, \psi$	10^3	591	236	250
Hydraulic masses	$\mu = \alpha\, \beta^{9/4}\, \theta\, \zeta^{-9/2}\, \eta^{-7/4}\, \sigma\, \psi$	10^6	2.76×10^6	1.1×10^6	$(0.95\text{–}1.35) \times 10^6$
Grain diameter	$\delta = \beta^{3/2}\, \phi^{-2/3}\, \zeta^{-3}\, \eta^{-3/2}\, \psi^{2/3}$	100	9.14	7.0	6.8–7.2

Unfortunately, knowledge of the fundamental laws of bed movement in large streams is still unsatisfactory. Therefore, it is not possible to build quantitative models for large streams with fine sediment. In this case, only qualitative models producing qualitative results can be built; but even those qualitative results cannot be trusted unless it is possible to find in each model a number of qualitative checks such as those described by Mr. Brown in his excellent paper about "Development of Mississippi River Channel." The care with which statements in general are made in Mr. Brown's paper illustrates the unreliability of such models and the great danger of drawing erroneous conclusions from them. It is hoped that as a result of basic research investigators soon will find the fundamental laws needed for quantitative model studies in this field.

K. G. TOWER,[22] **JUN. AM. SOC. C. E.**[22a]—The "Foreword" to the admirable series of papers appearing in this Symposium could advantageously have been more lengthy and explicit. In explanation of the objective of the research the rather bald question is stated: "Does the prototype act as the model predicted?" This question will cause the uninitiated to query: How, then, if at this late date such a question must be asked, has the art and science of hydraulic model testing reached its present prominence? For all practical purposes, of course, with the exception of the relatively infrequent case where air entrainment or the pressure of the atmosphere (or a combination of both) may be critical factors, models operated at atmospheric pressures will give a thoroughly reliable solution to problems which, in many cases, if there is an alternative, are susceptible to but questionable mathematical analyses.

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[22] Associate Engr., U. S. Engr. Dept., Portland, Ore.

[22a] Received by the Secretary March 1, 1943.

With perhaps a few exceptions, those engineers who have had recourse to the hydraulic laboratory for the solution of a specific problem have been "sold" on the idea of model testing. They have seen the solution obtained in the laboratory verified by the prototype. So far as has been observed and checked by rough field measurements, qualitatively and quantitatively, for all practical purposes the prototype has performed as predicted by the model. The irrigation-canal check-drop investigation presented by Messrs. Warnock and Dewey is typical of the type of problem that has been solved repeatedly in the past to the entire satisfaction of all concerned, and upon which successful solutions the present excellent reputation and reliance in hydraulic model testing have been founded.

Thus, qualitatively at least, with the exceptions noted, the question of the "Foreword" need not be asked of hydraulic laboratory personnel. The question should be reworded to read: "To what degree do quantitative measurements recorded in the laboratory differ from those obtained in the field?" Viewed in the light of the restated objective of this research, the Symposium is seen to represent the first organized public expression of the manner in which hydraulic designers and laboratory technicians are striving toward the achievement of perfection. Those who have participated in developing the technique and interpretation of hydraulic model studies from an awkward adolescence to a confident maturity within the past decade can consider themselves as having engaged in a job well done.

The attainment of perfection in any activity is highly desirable, and is always praiseworthy, even in those instances where the activity itself may be of doubtful merit. It is hoped, however, that before those engaged in the direction of hydraulic laboratories definitely conclude to spend all of their available spare time and funds in an effort to perfect the interpretation of results obtained from model studies that thought first be given to furnishing the hydraulic designer with reliable basic data which will permit final and economic solutions of his everyday problems. In metaphor, before worrying about the twine in the binder, first learn how to plow.

An important task of the future is the elimination of the need for a number of the hydraulic tests of models accepted by many agencies at present as standard practice. Typical structures frequently constructed should be analyzed in the hydraulic laboratory in such a manner that the data obtained therein, when correlated with other modifying or confirming data from prototype measurements, would constitute a reliable guide to the hydraulic designer and eliminate the necessity for the continual repetition year after year of the construction and testing of models of structures which are nearly identical.

Spillway crests furnish possibly the best example of the case in point. It is a recognized fact that the highest discharge coefficients are obtained with crests properly designed and shaped to fit the lower nappe curve of a discharge sheet over a sharp-crested weir. Hydraulic laboratories should endeavor, at the earliest opportunity, to supplement information already available on the curve of the sharp-crested weir lower nappe and the entire matter should be placed on record in such a manner that no hydraulic designer in the future, by reason of the lack of adequate information, need be found recommending a spillway shaped, for example, similar to the one examined by Mr. Soucek and illustrated in Fig. 50 in his study of submerged flow, "Meter Measurements of Dam Discharge." Stilling basins and transitions at the entrances to conduits are additional structural features deserving critical and intensive examination for the purpose of design standardization and the elimination of routine hydraulic model testing. The use of the three structural features mentioned requires the greater part of the work involved in the hydraulic design of dams and control works, and more comprehensive information on them is needed daily at the drafting boards of organizations engaged in this work.

The suggested research program will have to be conducted principally in laboratories as there are insufficient suitable existing full-scale structures, even if all were investigated, to contribute an appreciable quantity of the necessary basic data. Those existing structures that are suitable, however, well may be examined for data to be used as a check of the models at one or more stages of the variations in them which will be required.

The fact that at this late date the meager store of basic hydraulic information makes the proposal contained herein necessary is sufficient evidence that considerable fortitude, initiative, and skill will be demanded of those participating in the program. The magnitude of the work required is such that the program will have to be deferred until after the war. In the meantime, the practicing hydraulic engineer will welcome any prototype study that contributes to his working knowledge regardless of whether or not the intent of such study is merely to confirm or deny data obtained from a particular hydraulic model.

R. J. Pafford, Jr.,[23] Esq.[23a]—Individually the papers are an interesting and enlightening addition to the literature on hydraulic model practice and hydraulic design, for which the various authors are to be commended. Collected together in this Symposium, they represent important contributions toward more effective and more comprehensive use of the hydraulic model as an instrument of design, through the illustration of the wide field of problems susceptible to model analyses, and toward better general understanding of how this tool can be improved further by the illustrations of both its capabilities and its present limitations.

Considering the subject matter of the Symposium as a whole, the paper by Messrs. Warnock and Dewey is particularly thought provoking. There is urgent need for much more prototype data, both for evaluation of accuracy of the model and for direct use in design. This calls for the creation of greater interest on the part of all concerned, and for still greater cooperation in overcoming the difficulties of making prototype measurements. Past lack of general interest undoubtedly has been contributed to by both the model worker and the designer of hydraulic structures. Too great enthusiasm of the model worker arising from his intense interest in his tool in many cases has tended to "oversell" the capabilities of the model; and also, in many cases, there has been a tendency of the designer to present his finished product as if it were the best and final answer to a given set of conditions rather than just a good design which will perform satisfactorily, but which undoubtedly could be improved either in performance characteristics, economy, or both, if better information were at his disposal. A more realistic attitude on the part of both can contribute materially to greater general interest and, in turn, to greater opportunity for the collection of needed prototype data, even where the expenses of observations may be high.

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[23] Engr. (Hydr.), U. S. Engr. Office, Omaha, Nebr.

[23a] Received by the Secretary March 1, 1943.

The numerous difficulties involved in obtaining accurate prototype data make this indeed an exasperating and expensive field, yet fascinating, in the writer's experience at least, because of these very difficulties. The utmost in ingenuity and the fullest possible interchange in ideas are essential. With reference to difficulties with equipment such as that used in measuring spillway bucket velocities and water-surface elevations over spillways as mentioned by Messrs. Warnock and Dewey, the following suggestions are offered for general consideration in future cases of this nature. Tubes leading to observation galleries, in many cases, can be kept open by regular flushing with clear water at frequent intervals from the time of installation until all desired performance data are collected. This often will be a nuisance to operating personnel, and of course in some instances will be impracticable; but then, why install the tubes if they are to become unserviceable before data can be collected? There also seems a possibility that small tubes placed without sharp bends, with a flexible corrosion-resisting wire to be removed only during tests extending their full length (same idea as is used in an automobile hand-throttle control except that the wire would be pulled entirely from the tube during tests) might solve this clogging problem at some locations. The problem of coating or plugging of pitot-tube openings is not necessarily beyond solution either. It would seem that some kind of protective cover which might be removed either directly or by remote control just prior to a test could be devised for many installations. Turning to the problem of measurements of water surface of the flow over spillway sections of a dam, it appears that for at least some cases not complicated by spray from gates at partial opening, some relatively inexpensive method employing a narrow intense light beam and transit observations of the "spot" projected on the water surface (possibly coupled in some cases with measurement of reflection angles) could be utilized. Developments along the lines of sonic or electronic soundings also would seem to hold considerable promise for the future, perhaps especially in cases complicated by spray and foam on top of the "solid" jet.

As mentioned in the Warnock-Dewey paper, still and motion pictures of all flow conditions observed both in laboratory and field are indeed invaluable for purposes of comparison of model and prototype performance. On this score, the writer desires to emphasize the desirability of complete pictorial records of model performance from viewpoints that can be duplicated in the field as well as from vantage points possible in the laboratory which may better illustrate performances of certain features of the model but which are impractical in the field. It has been the writer's unfortunate experience, which is feared not unique, to fail on several occasions in attempts to compile fully satisfactory comparative pictorial records because the only laboratory pictures were taken through the sides of glass-walled flumes or from points which would require that the prototype photographer be suspended in mid-air over the stream below a dam. On the other hand, really excellent photographic comparisons are possible, as illustrated by Fig. 1, when model photographs from proper angles are available. For most effective photographic comparison of model and prototype performance, attention to relative focal lengths of camera lenses used in the laboratory and in the field, to insure similar perspectives of the photographs, is almost as important as the position of the camera.

In 1942 the writer had a convincing demonstration of the qualitative accuracy of the hydraulic model in a field not covered by the papers comprising the Symposium. In connection with design of a spillway stilling basin utilizing baffle piers for energy dissipation, there was some concern for a possible cavitation hazard, and extensive model investigations were made under the writer's general supervision in two leading hydraulic laboratories. The test method in one involved the measurement and analysis of pressures that occurred on baffle surfaces in a model under atmospheric pressure; the other involved visual inspection and interpretation of the performance of a model erected in a vacuum tank apparatus which enabled the reproduction of pressures in the correct scale ratio. To enable the best possible interpretation of the tests for the structure being designed, both laboratories first conducted tests on models of an existing stilling basin also dependent on baffle piers for energy dissipation where some pitting in certain areas along the sides of the baffle piers had actually occurred during the passage of an extreme flood. So far as the main test program was concerned, it is sufficient to state that the results of independently conducted tests at each laboratory were in reasonable agreement with conditions observed at the existing prototype and with conditions indicated at the other laboratory, and that a design involving partly streamlined baffle piers was evolved. The original descriptions and photographs of conditions at the existing structure had indicated pitting only along certain areas on the sides of the baffle piers. During testing of the model of this structure in the vacuum tank, it was seen that conditions along the stilling basin floor in certain areas between the baffle piers were apparently nearly as severe as along the sides of the baffle piers where pitting had occurred and there were also indications of incipience of cavitation at the extreme upstream edge of the tops of the baffles. Incidentally, indications of areas on the sides of the baffles in which cavitation might be suspected in the model were in excellent general agreement with pitted areas in the prototype. Some time after observing the foregoing phenomena at the laboratory, the writer visited the existing prototype for the specific purpose of further investigation of the extent of pitting in the stilling basin area. Visual inspection revealed a minor degree of pitting at the leading edge of the tops of the baffle piers in the areas indicated by the model. The stilling basin floor was covered by a thin layer of mud and

4 or 5 ft of murky water, making visual inspection there impracticable at the time. Careful probing, however, definitely established that there was considerable pitting of the apron floor in the very areas indicated by the model. As nearly as could be determined by "feel" through the sticks used for probing, the degree and depth of pitting were very similar to the pitting on the sides of the baffle piers where the depth ranged up to 2 or 3 in. below the original concrete surface.

EDWARD H. SCHULZ,[24] M. AM. SOC. C. E.[24a]—The physical comparisons of models in this Symposium cover a wide range of structures, including irrigation works, discharge outlets, dry dock suction chambers, lock construction, dam outlets, discharge and pressure heads, spillway coefficients, meter measurements, river improvement, and lock approaches.

There are discussions of qualitative and quantitative comparisons, the latter of which were found to be the more valuable and important. The ratio of prototype to model has a wide range from 1 : 5 to 1 : 600. Hydraulic models should be on as large a scale as reasonable. Perhaps there has been too little interest in comparing performance of the prototype with the model, partly due to the difficulty of making measurements. The accuracy of measurements is important in the prototype as well as in the model. Some of the more difficult determinations are slope, velocity, discharge, pressure, friction, etc., and these are more difficult in closed circuits than in open ones, and more important for the prototype than for model studies. Studies as to erosion, bar movement, etc., may be necessary.

Geometric similarity is very important. The comparisons should cover the range of hydraulic conditions. The cost of prototype field measurements may be a deterrent.

Conclusions in the Symposium, based on the tests of various structures, show that performances generally have been in line with the model tests. Such tests are an excellent help to the engineer. Model conditions, in general, were substantiated by the prototypes, especially where geometric similarity existed. Discrepancies were within 5%, and probably less than would have occurred without such model tests. There were some anomalous results, however, such as pressure head comparisons. The meter measurements of dam discharge of the model were about 5% less than those of the prototype. The general agreement was satisfactory. On the river improvement the prototype reproduced the results indicated by the model. Some diversion was to be expected. On the lock approaches there was satisfactory verification.

In conclusion, the writer considers model tests very useful in the study and correction of hydraulic relations and conditions. In building any structure where such studies and comparisons are likely to be needed and where the expense is justified, provision should be made to incorporate appropriate accessories and equipment in the prototype during construction. Measurements can then be made more readily and tests can be conducted at any time within the limits of the structure and the range of the hydraulic conditions to be expected.

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[24a] Received by the Secretary March 3, 1943.

F. T. MAVIS,[25] M. AM. SOC. C. E.[25a]—In a singularly complete manner Mr. Soucek has presented the results of his laboratory studies and their comparisons with observations of a prototype. The systematic relationship between relative discharge (submerged to free) and submergence shown in Figs. 52 and 53 reflects the precision of the laboratory measurements. It is clear from Fig. 53, as Mr. Soucek states in his first conclusion, that generalizations of the effect of spillway profile and absolute discharge should be accepted with caution as a basis for estimating precisely the relationship between headwater head, submergence ratio, and discharge over a spillway.

Profile of Model *B*, in Inches

x	*y* Below Crest	
0"	11.25" and 1.45"	
2	0.25	
4	0.00	Crest
6	0.30	
8	0.75	
10	1.85	
12	3.00	
14	4.30	
16	5.75	
18	7.55	
20	9.95	
21	11.25	Toe

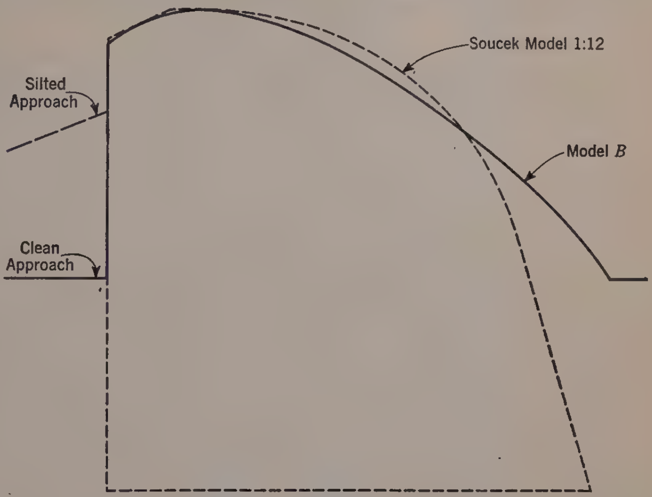

FIG. 73.—COMPARISON OF MODELS

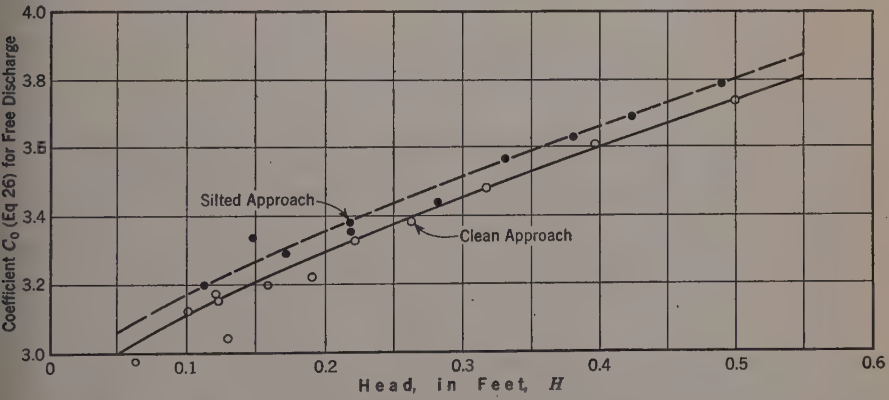

FIG. 74.—TESTS OF MODEL B; FREE FLOW

The verification by laboratory test that the relation between submergence and discharge was unaffected by the condition of the nappe, whether plunging or flowing, should dispel misgivings for one shape of spillway at least. An interesting problem of hydromechanics may lie behind the data plotted in Fig. 54.

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[25a] Received by the Secretary March 5, 1943.

In 1934 the writer conducted a brief series of tests at the University of Iowa, Iowa City, Iowa, on a model 11.25 in. high, designated Model B, in Fig. 73. The model profile discussed by Mr. Soucek is shown for comparison.

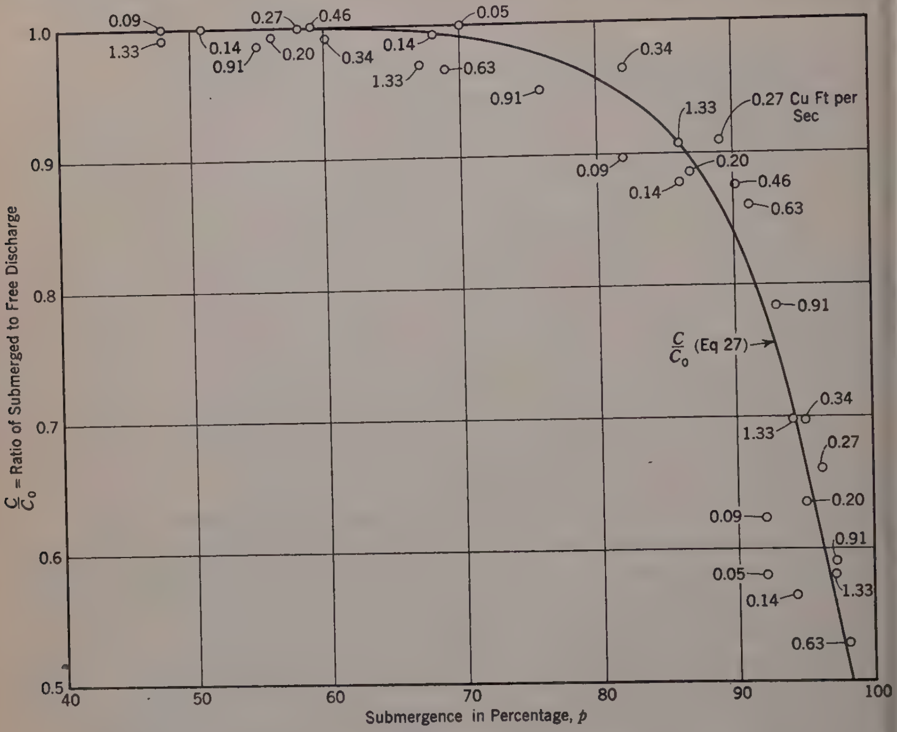

FIG. 75.—EFFECTS OF SUBMERGENCE; MODEL B

Model B was tested in a glass-walled flume 12 in. wide under two conditions—"clean approach" with a level approach channel; and "silted approach" with a sand fill 1 on 2.5 intersecting the upstream face of the model 4 in. below the crest.

Fig. 74 shows the relation between coefficient for free discharge

$$C_0 = \frac{Q_0}{L\,H^{1.5}} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and head for both "clean approach" and "silted approach." Analysis of data on this basis is simpler than analysis based on Eq. 13. If a model is truly representative of its prototype, either analysis leads precisely to the same conclusion. Used discretely either analysis leads to a valid engineering conclusion.

Fig. 75 shows the relationship between submergence percentage, p, and the ratio of submerged to free discharge as expressed by the formula

$$\frac{C}{C_0} = 0.01\left[100 - 2^{0.2(p-70)}\right] \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Opposite each plotted point is a figure representing the corresponding discharge, q, in cubic feet per second per linear foot of crest. The absolute rate of discharge, q, may be recognized in Fig. 75 as an independent variable. Although the data plotted in Fig. 75 might be presented in support of Mr. Soucek's discussion of Fig. 53, one may ask whether the ultimate uses of the data warrant the refinement of analysis in this instance. To explore this point the generalized analysis of data from tests of Model B were compared with data reported by Mr. Soucek in Cols. 2 and 3 of Table 9.

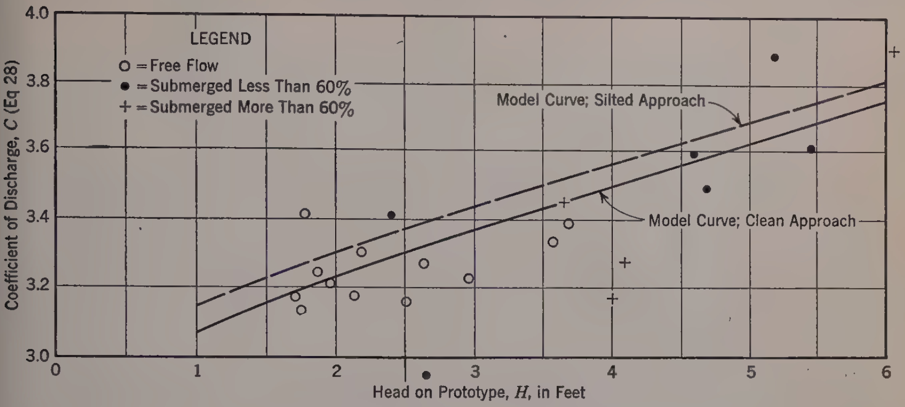

FIG. 76.—OBSERVATIONS OF PROTOTYPE

Fig. 76 shows the coefficient of discharge for the prototype:

$$C = \frac{Q}{L\,H^{1.5}} \qquad (28)$$

as a function of the observed headwater depth, H. Observed data are classed in three categories—free flow, submergence less than 60%, and submergence greater than 60%. Fig. 76 shows also the curves based on tests of Model B for clean approach and silted approach. The conversion from model to prototype was based on Froude's law. Note that the scatter of points in the first category (free flow) is of the order of 5% of the coefficient of discharge. According to Fig. 53, the submergence would have to exceed 80% before the actual discharge would be reduced correspondingly below that calculated for free discharge. Hence, as a tentative premise one may assume that only the last three field observations in the third category (Table 9) are significantly below a valid general curve relating coefficient of discharge and head on prototype as shown in Fig. 76. Tentatively, then, the relationship between head and free discharge for Model B and the prototype seems verified by Fig. 76.

As stated, Eq. 27 in Fig. 75 shows an average curve relating the ratio of submerged to free discharge $\frac{C}{C_0}$, and percentage submergence, p. Although this equation appears complex, it means that the ratio of submerged to free discharge, $\frac{C}{C_0}$, is reduced from unity by 1.0% if the submergence is 70%; this percentage reduction is successively doubled for each additional 5% of submergence. For example,

For a submergence ratio (%) of:	the reduction of $\frac{C}{C_0}$ is	and the ratio $\frac{C}{C_0}$ is
70	0.01	0.99
75	0.02	0.98
80	0.04	0.96
85	0.08	0.92
90	0.16	0.84
95	0.32	0.68

As a final step, the writer compared observed data on the prototype reported in Table 9 with corresponding data computed on the basis of tests of Model B for both free and silted approaches. Adjusted discharge (Col. 2, Table 9), headwater depth (Col. 3), and tailwater depth (Col. 6), were used as the basis for a spillway 273.50 ft long. The maximum difference between field observations and corresponding calculations based on tests of Model B with the generalized submergence correction (Fig. 75) was 11% for the model with clean approach and 13% for the model with silted approach. These values correspond to a maximum difference of 15% in Table 9. The mean absolute percentage difference between Model B and the prototype was 3.8% for the clean approach and 4.2% for the silted approach. These data correspond to a mean absolute percentage difference of 6.0% in Table 9.

The conclusions seem justified, therefore, that—

(*a*) The differences in model spillway profiles shown in Fig. 73 are not significant in reflecting the behavior of the prototype under conditions of free and submerged discharge; and

(*b*) The generalized relationship between submergence percentage, p, and the ratio of submerged to free discharge, $\frac{C}{C_0}$, as expressed by Eq. 27, is valid within the scatter of field observations on the prototype.

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(42a) **"Weir Experiments, Coefficients, and Formulas,"** by R. E. Horton, *Water Supply and Irrigation Paper No. 200*, U. S. Geological Survey, 1907. (Original source: "Report of the U. S. Board of Engineers on Deep Waterways," 1900, Pt. I, p. 291.)

(62) **"Die Internationale Rheinregulierung von der Illmündung bis zum Bodensee,"** by Karl Böhi, *Schweizerische Bauzeitung*, Vol. 109–110, 1937, p. 187.

(63) **"Formulas for the Transportation of Bed Load,"** by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 561.

(64) **"Beitraege zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen fuer Stroeme, Kanaele und geschlossene Leitungen,"** by A. Strickler, Mitt. des eidg. Amtes fuer Wasserwirtschaft, No. 16, Bern, 1913.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

EARLY CONTRIBUTIONS TO MISSISSIPPI RIVER HYDROLOGY

Discussion

By C. S. Jarvis, M. Am. Soc. C. E.

C. S. Jarvis,[31] M. Am. Soc. C. E.[31a]—Mr. Davenport has contributed an interesting, thought-provoking discussion, which discloses specifically why the most earnest attempts at evaluation and interpretation of early discharge records are under such a handicap, in the judgment of those best equipped to carry the project forward or to encourage and guide other investigators. Against all the implications that such a task cannot be accomplished with even a fair degree of reliability, only one answer is ventured:

> Every independent, unbiased, logical approach toward the evaluation and coordination of those early records pays a tangible dividend, either by establishing, confirming, or revising usable estimates and approximations. We are surely nearer to the truth now than we were before any sustained attempts were made at evaluation. Further investigations and revisions are invited. Thus far, many confirmations have resulted from various approaches.

Acting in conformity with basic legislative and administrative authority, Humphreys and Abbot collected available pertinent information concerning flood heights, crevasses, levee construction, and dimensions on the Mississippi River,[32] "and well authenticated changes in the banks of the river, etc. * * *. In each set of cross-sections, the velocity of the current was measured—in some instances, with great elaboration." In view of such items of record, exception must be taken to the concept expressed by Mr. Davenport, implying that practically the sole authority for discharge and cross-sectional data, as well as for the rating curve for the early years of record, was the 1858 series of discharge and gage readings at Vicksburg, Miss., and Natchez, Miss. As

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Note.—This paper by C. S. Jarvis, M Am. Soc. C. E., was published in March, 1942, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: September, 1942, by Messrs. Robert Follansbee, and R. W. Davenport; November, 1942, by Charles Senour, M. Am. Soc. C. E.; December, 1942, by John C. Hoyt, M. Am. Soc. C. E.; and February, 1943, by Messrs. George O. Guesmer, and H. T. Cory.

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[31a] Received by the Secretary January 26, 1943.

[32] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 2.

opposed to such a theory, note the comparison of methods used for float measurements in 1851 and 1858 and, also, the unsatisfactory tests of Saxton's current meter.[33] Some of the following references prove that fragmentary data were collected prior to 1850 (some as early as 1838), and that the results were available for incorporation in the final report.

Among the many references that establish the broad scope of investigations included in the Humphreys and Abbot report, the following have been selected:[34]

> "The corresponding velocity, taken from a table constructed for the purpose, was considered to be the mean velocity of the division—absolute for the observations of 1851, and relative for those of 1858. For the shore divisions, unless the floats happened to be well distributed through them, the mean velocity was assumed to be eight-tenths of that at the outer edge, a rule deduced from a subdivision and study of the velocity, when thoroughly measured in these divisions [200 ft apart]."

Also in this report, included in George G. Meade's letter,[35] is a brief description of his extensive observations relating to current velocities, directions, and vertical distributions in and around the delta channels during the year 1838; and there is a reference[36] to the 90-ft cord used for connecting the surface float with the submerged float. The location[37] of the maximum velocity in a depth of 100 ft is given as 30 ft from the surface—a fair approach to what is acceptable today.

Furthermore, there are numerous references to records and measurements conducted by Lt. Robert A. Marr, U. S. Navy, Prof. C. G. Forshey, M. Am. Soc. C. E., and Charles Ellet, Jr., during years earlier than 1850 (for example, Lieutenant Marr's record of daily rainfall, gage heights, temperature, and river discharge at Memphis, Tenn., together with weekly evaporation, April 1 to July 15, 1848,[38] river data, March 1, 1850, to March 1, 1851, and related activities[39]).

After describing the discharge and other measurements on the Lower Mississippi River from February, 1851, to February, 1852, and from December, 1857, to December, 1858,[40] Humphreys and Abbot declared, "Besides these continuous series, many measurements of width, depth, area of cross-section, discharge, etc., were made both upon the Mississippi and upon its tributaries and bayous from the Ohio to the Gulf." It seems reasonable to assume that all such data underlay the discharge determinations for the 33 years listed in Table 1. These values are of acceptable quality, as proved repeatedly by various approaches described hereafter.

Even though the scope of the statement concerning other measurements outside the continuous series may apply mainly to the one decade covering the Humphreys and Abbot assignment, the aforementioned and associated refer-

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[33] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 234.

[34] *Ibid.*, p. 235.

[35] *Ibid.*, p. 507.

[36] *Ibid.*, p. 664.

[37] *Ibid.*, p. 661.

[38] *Ibid.*, p. 201.

[39] *Ibid.*, pp. 129, 202, and 378–394.

[40] *Ibid.*, p. 230.

ences indicate that it is applicable in some degree for the years preceding 1850. Thus, the persistent attempts to evaluate the tributary, main channel, and crevasse discharges, and to correlate these quantities at successive stations from Columbus, Ky., to Carrollton (New Orleans), La.,[41] unmistakably indicate the avidity with which the authors incorporated basic data and extended their use within practicable limits. Unquestionably, some of those segments of gage-height hydrographs for the Natchez, Vicksburg, and New Carthage, Miss., stations (the latter lying between the first two),[42] for 33 years, 1818 to 1860, were associated with series of discharge determinations, by means of floats; either natural—driftwood and uprooted trees—or artificial—weighted rods or multiple floats—as noted in various pages of the text. Otherwise, the separate determinations of discharge by Lieutenant Marr, Professor Forshey, and Mr. Ellet cannot be accounted for nor accredited reasonably.

The rating curve for the earliest biennium of record at Carrollton, 1851 and 1852, depended on more than 100 separate gagings, of which 21 were definitely dated on the chart. There can be no reasonable doubt that the earlier fragmentary or detached data were incorporated in some manner to make the rating curves more nearly representative of conditions during earlier years. Otherwise, it would be difficult to account for the close agreement in the last two columns of Table 1, showing 33-yr means of 614,000 cu ft per sec and 617,000 cu ft per sec, a difference less than 0.5%, even though the means for the last 10 years of the same tabulation differed by nearly 2.0%. It would be gratifying, indeed, if one could be assured that stream-discharge determinations under the most up-to-date, standard practice could stay within such close limits, or that the separate records of earlier periods were within limits five times as great.

Fig. 5 is of unusual interest because it follows the general form of patterns set in earlier years by the Mississippi River gagings at Natchez, Vicksburg, and Warrenton, Miss., beginning in 1858; at Columbus, in 1857; and at Carrollton, as early as 1851. For the last-named station, well-defined segments of rating curves are available for twenty-one separate years or groups of years between 1851 and 1929. They all lie within the limits of those curves selected for inclusion in Fig. 3 for Carrollton and are numbered chronologically from the earliest to curve 6, the rating curve for 1929. With few exceptions, the successive patterns at all gaging stations on the Lower Mississippi River show deviations of more nearly ± 5% as against ± 10% in the positions of rating curves near El. 34 (Fig. 5). The most outstanding exceptions are afforded by the records of river gagings at Columbus, for the years 1857–1858, and 1881–1882, showing somewhat greater variations than ± 10%. This pronounced variability persists in diminishing degree at the same station to include the year 1937, according to the writer's interpretation of official data, published mainly by the Mississippi River Commission. This analysis seems to provide a tentative answer to the query, for which Mr. Davenport admitted he had none, as to how nearly typical was the variation in plotted gagings for the year 1938 at Vicksburg. Fig. 5 typifies the shifting of rating curves for the more erratic periods, but not for the maximum variability.

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[41] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, pp. 128–132, 182, and 372.

[42] *Ibid.*, Plates V, VI, and VII.

Recent official records for the Lower Mississippi River stations disclose consistent relationships generally, but occasional departures of 5% or more for gagings by different agencies (or even by the same agency) with the river surface elevation practically constant.

1. Would it seem inconsistent to ascribe some portion, possibly approaching one half of the aforementioned 20% variation for the gagings in Fig. 5, to both the personal equation of the operators and the inherent defects or erratic movements of current-meter vanes due to suspended silt and fine granular material or other debris encountered in all such rivers, but never in the standard rating flumes of laboratories?
2. With all the refinements now applied to the measurements of depths at intervals along the cross section, what uncertainties still persist because the heavy sounding weight settles into the mud or silt of the stream bed or because it encounters either a ridge or a valley?
3. Is the square foot, or portion thereof, occupied by the current meter for the purpose of determining velocity of flow actually representative of the thousands or tens of thousands of such elementary areas within the cross section for which it serves as a sample?

In view of such unsolved problems as well as the shifting of currents and bars within a cross section and in view of the pulsations of waves or unpredictable movements of eddies in the course of turbulent flow (with all other uncertainties inherent in stream gaging on deep alluvium even where the best practical standards prevail), it seems noteworthy that such consistent relationships have been developed and maintained between river stage and discharge.

No less amazing are the results achieved normally in connection with the early gagings of the Mississippi River system without the conveniences afforded by fixed bridges (being dependent upon floating equipment and shore installations or observations, as well as extra labor and ingenuity) to accomplish the difficult assignments under pioneer conditions. Surely, some compensations must have accrued from the direct measurements of stream velocities by weighted rods, multiple floats, or uprooted trees. This procedure integrated and averaged the movement within much larger samples of cross-sectional area, and furthermore avoided the inherent approximations of rating tables for current meters and the recognized uncertainties involved in their practical use. There is some reassurance in the observation that velocities are determined by the use of floats to this day, when circumstances warrant, either for flood crest or for routine gagings; and results are duly published along with those obtained by the use of current meters, with symbols to indicate whether one or two meters, double floats, rods, or surface floats were employed. Many current-meter gagings have had the advantage of float measurements to test the action of the meter vanes, even though these experiments were not necessarily included in the published reports.

Mr. Follansbee has contributed some very interesting historical notes relating to the development of the art and science of stream gaging.

Mr. Hoyt has written his impressions, based on a much broader experience than comes to most. With deep gratitude the writer acknowledges the encouragement accorded by Mr. Hoyt, particularly in the projects for evaluation and comparison of flood data pertaining to foreign rivers, such as the Nile, as well as to American rivers, especially those with the longer records.

Mr. Senour has contributed a number of important items that would be difficult to obtain outside the central office of the Mississippi River Commission. Fig. 6 provides an authoritative answer to many queries that have been directed toward the writer, as to the stability of stage-discharge relationships at the lower stations. Because the 4,900 sq miles of intervening drainage area between Vicksburg and Natchez represent only 0.43% of the total, although the proportionate runoff was more nearly 1%, the data at each of these stations have served at times for both, with the single percentage differential taken into account wherever circumstances warranted. Inclusion of the complete data for 1937, which were not available when Fig. 2 was prepared, provides an added rating curve, as described by Mr. Senour, to the right of curve 6 for Vicksburg. For 1941 the rating curve is still farther to the right, according to the U. S. Geological Survey, presumably measuring the increase of channel capacity resulting from cutoffs. The Natchez gage readings were about 5 ft higher for equal discharges, traceable to a lower datum with respect to hydraulic slopes for successive stages.

The three crest stages for Carrollton for the years 1832, 1840, and 1844, to which Mr. Senour takes exception, were merely transcribed along with associated data in Table 3 from *Professional Paper No. 13*.[43] If the several curves of Fig. 3 pertaining to Carrollton gagings are plotted or superimposed on Fig. 7, these rating curves will be found to cover approximately one half of the shaded area along the central strip. This shows that the variations among single observations are nearly double the range occupied by rating curves. When it is realized how nearly the extreme variations recorded in Fig. 5 equal those in Figs. 6 and 7 (the latter two covering 90 years instead of the single year 1938), the factors of the personal equation of operators and the practical limitations on the accuracy of current-meter gagings seem to demand consideration.

In response to Mr. Senour's request for clarification of data presented in Table 7, it should be noted that the writer aimed to record the progressively increasing concentrations of flood water associated with peak discharge and aggravated soil wastage, including both the confined channel flow and the well-defined storage or lateral floodway capacities then in use. Examples of this concept were reported in 1929 by C. W. Kutz, M. Am. Soc. C. E.,[44] as 2,662,000 cu ft per sec at the latitude of Arkansas City, Ark., in April, 1927, or 2,350,000 cu ft per sec near Old River, La. At the same time, General Kutz (then Colonel) dealt with possible or probable maximum floods at Cairo, Ill., ranging from 2,250,000 cu ft per sec to 2,400,000 cu ft per sec. According to George R. Clemens,[45] M. Am. Soc. C. E.:

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[43] "Report on the Physics and Hydraulics of the Mississippi River," by A. A. Humphreys and Henry L. Abbot, *Professional Paper No. 13*, Corps of Engrs., U. S. Army, 1876, p. 443.

[44] "The Work of the Mississippi River Commission," by C. W. Kutz, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 707.

[45] "The Mississippi Meets the 1937 Flood," by George R. Clemens, *Civil Engineering*, June, 1937, p. 381.

> "Thus the Mississippi River met the first thrust of the 1937 flood. Cairo was the only dry city in the Ohio basin. The New Madrid floodway functioned essentially as planned, and protected not only Cairo but the entire St. Francis basin from a flood that most certainly would have overtopped the levees as they stood in 1927. At the crest, about 2,000,000 cu ft per sec passed Cairo, and about 500,000 cu ft per sec was carried in the floodway."

The writer's concept was that the same 500,000 cu ft per sec could be regarded as in temporary storage or detention between Cairo and Memphis, Tenn. The plan sponsored by the Chief of Engineers to[46] "provide for a flood which without reservoir control would reach 2,600,000 cubic feet per second in the Mississippi between Cairo and the Arkansas" was based on both a knowledge and a thorough understanding of the data including the 1937 flood rise, the well-organized defense of the Cairo levees, the relief afforded by the access to the New Madrid floodway, and the potentialities of future floods.

Another reference which seems to support the concept of total discharge from both confined channel and floodway or lateral storage is to be found in an article by Maj. Raymond G. Moses:[47] "At the crest, of course, the Arkansas-White backwater area was still taking water but the flows from those two streams were sufficient to increase the flow in the main river to about 2,150,000 cfs at the crest at Arkansas City * * *."

The tabulated discharges for the years 1909, 1916, 1920, and 1928 are weighted means of either official or semi-authoritative data. Official quantities including the latest revisions should be preferable.

Mr. Guesmer not only observed some of the initial investigations of Mississippi River discharge data, but also participated to the extent of locating some of the most valuable folios and summaries of unpublished hydrologic data, of which the writer had no previous knowledge. It seemed logical to deal first with the headwaters and then with the successive tributaries and main river stations. The procedure thus developed was found to be very satisfactory and capable of deriving successive discharge quantities generally within

TABLE 9.—DISCHARGES, IN CUBIC FEET PER SECOND, FOR THE DESIGNATED YEARS OR OTHER PERIODS

Description	Latest 10 yr	5 yr 1934–1938	1937–1938	1936–1937	1935–1936	1934–1935	1933–1934
271							
Derived	55,163	52,701	53,458	57,312	59,320	53,216	40,197
Departures (numerical)	254	263	−1,500 748[a]	−828	290	−824	417
Departures (%)	0.5	0.5	1.5	−1.5	0.5	−1.6	1.0
272							
Observed	54,909	52,438	51,210	58,140	59,030	54,040	39,780

[a] Initial filling for storage in reservoir was 1,091,000 acre-ft, representing an inflow of 1,500 cu ft per sec for one year.

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[46] "Comprehensive Flood-Control Plan for Ohio and Lower Mississippi Rivers," *Committee Document No. 1*, House Committee on Flood Control, 75th Cong., 1st Session, April 6, 1937, p. 6, paragraph 20.

[47] "High Water in the Lower Mississippi," by Raymond G. Moses, *The Military Engineer*, May-June, 1937, p. 167.

5%, and often within 2%, as illustrated in Table 9 for the Tennessee River at Savannah, Tenn.

The initial reckoning disclosed an unusual departure of nearly 4.5% for the water year 1937–1938 until it was discovered that the initial filling of the reservoir accounted for the withdrawal of 1,500 cu ft per sec from the stream flow. In the same manner, it was quite a simple problem to derive the yearly and periodic mean discharges for Cincinnati, Ohio. For this purpose, the observed discharges of the Ohio River at Huntington, W. Va., were correlated with records of the gaged tributaries and estimated yields for the ungaged areas. The latter were either equal to or somewhat less than the observed discharge per square mile of the gaged tributaries. For any of the intermediate stations on the Mississippi River and its major tributaries, or even the minor branches, this procedure seemed to render acceptable values.

Various tests showed that the ungaged portions of valley floors ordinarily yield somewhat less than do equal areas in steeper and more elevated regions. Furthermore, it is generally known that the larger tributaries head on the higher ground; therefore it is customary to estimate the yield per square mile of ungaged areas, including minor tributaries, as 80% or 90% of the unit yield for the adjacent gaged tributaries. Observed and derived discharges for 56 stations selected from among the 400 or more available are listed in Table 8.

The progressive summations, comparisons, samplings, and adjustments which led to these results for all important stations within the Mississippi River basin involved the following principal phases:

1. Assembly of outstanding and representative precipitation records;
2. Interpolations from neighboring stations to fill all gaps;
3. Comparisons of monthly or yearly means for the group of stations with the total record means to obtain excess and deficient depths for successive months or years;
4. Computations of temperature departures from normal in the same manner, for one or more representative stations;
5. Computations of runoff departures from record mean at the selected lower river station (record-mean annual runoff depth minus successive yearly depths);
6. Comparison of rainfall and runoff departures in the light of temperature departures from normal;
7. Investigation of particular storm patterns in all cases where rainfall and runoff departures are greatly at variance or of opposite signs;
8. Derivation of discharge at the lower river station from summations of tributary discharges and estimates of ungaged areas based thereon;
9. Interpretation of hydrographs in the light of all previously assembled data and their established relationships; and
10. Checking the arrays of data by the use of weighting factors, used in multiplying the different tributary discharges in order to build up the approximation to the lower station discharge, year by year, month by month, or day by day, if time factors, valley storage, and flood routing are all taken into account.

Weighting Factors.—The concept and use of weighting factors may be explained best by a practical example. In the quest for suitable samples of tributary flow that may be used as an index of lower station discharge, the essential criteria for tributary records include not only their length, continuity, and quality, but also their degree of correlation with discharge at the lower station. The Gasconade River at Rich Fountain, Mo., is outstanding for such correlation; the Kanawha River at Kanawha Falls, W. Va., has a record of outstanding length and reliability. Their respective drainage areas of 3,180 sq miles and 8,367 sq miles, totaling 11,547 sq miles, represent nearly 1% of the 1,144,500 sq miles drained above Vicksburg, and the average yield is 2.667% of the total. Evidently, an approximation to the annual discharge at Vicksburg is afforded by taking $\frac{100}{2.667} = 37.5$ times the recorded discharges at Rich Fountain and Kanawha Falls, according to the normal procedure for the "sampling method."[48] The weighting-factor development came in response to the query, "Would it be more satisfactory to accord different weights to the tributary drainages, calling for a greater factor for the Gasconade River and a lesser one for the Kanawha River, or vice versa?" Adopting the water years 1939 and 1940 for the observation or test period, the simultaneous equations become: For 1938–1939,

$$2{,}787\,x + 10{,}090\,y = 572{,}600 \quad \ldots\ldots (1)$$

and for 1939–1940,

$$1{,}611\,x + 10{,}940\,y = 332{,}500 \quad \ldots\ldots (2)$$

$$\frac{2{,}787}{1{,}611} \times \text{Eq. } 2 = 2{,}787\,x + 18{,}926\,y = 575{,}219 \quad \ldots\ldots (3)$$

$$\text{Eq. } 3 - \text{Eq. } 1 = 8{,}836\,y = 2{,}619 \quad \ldots\ldots (4)$$

$$y = 0.296401 \quad \ldots\ldots (5)$$

Substituting the value of y (Eq. 5) in both Eq. 1 and Eq. 3—$2{,}787\,x = 569{,}609$; and $x = 204.3807$.

This disparity in weighting factors results in almost complete dependence on the smaller tributary, outstanding indicator of Vicksburg discharge.

Use of the foregoing weighting factors for the Gasconade River and Kanawha River annual discharges for the 20-yr period, 1921 to 1940, resulted in a maximum departure of + 46.3%, with a mean absolute of ± 21.0%, as recorded for Item 17, Table 10. According to data under Item 91 (Table 10*B*), the substitution of discharge data from the Greenbrier branch of the Kanawha River, draining only 1,357 sq miles, with the Gasconade area, representing only 0.4% of that at Vicksburg, resulted in new weighting factors of 204.9578 and 25.756673, respectively. Under a 20-yr test, 1922 to 1941, these factors proved to be more satisfactory than the first pair, as the maximum and mean-absolute departures were reduced to − 42.4% and 15.1%, respectively, for the Mississippi and Atchafalaya rivers. However, Item 22 indicates what may be attained by such tests with 87.2% gaged, resulting in reduced percentages of + 10.2% and 3.6% over a 22-yr checking period. Such percentages approach so nearly the attainable accuracies in river gaging on alluvial formations that they seem to attest the consistency and reliability of the basic data underlying those derivations or estimates; furthermore, they may constitute a proof or indorsement of the derivation and testing method employed. Similarly, in Item 7 (departures of − 23.9% and 9.6% for 65 years), in Item 32 (departures of + 20.4% and 7.9% for 52 years), and in Item 67 (departures of + 18.2% and 5.7% for 52 years), the fairly close agreement between derived and observed discharge data extends beyond the range of mere coincidence.

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[48] "Estimating Annual Yield from Ungaged Drainage-Basins and Effect of Land-Use Treatment on Surface-Runoff," by C. S. Jarvis and H. C. Murto, *Transactions*, Am. Geophysical Union, 1941, pp. 875–893.

The fact that some other series of weighting factors, similarly derived, bring more discordant results would seem to indicate that either the test years or the individual tributary discharges were less representative. For example, the aforementioned Kanawha River discharges showed a larger quantity for the lean year (1939–1940) than for the average year (1938–1939) at Vicksburg, and, therefore, the resulting weighting factor is small, and shifts most of the burden to the smaller Gasconade River.

Least Squares Solutions.—Items 102 to 107 in Table 10*B* and Items 45 and 46 in Table 10 were solved by the method of least squares. The obvious advantage of this approach is that the observation period may be extended beyond the number of years that corresponds to the number of unknowns. Actually, the practical application of the simplest algebraic approach was found to accomplish the same results. For example, a 20-yr period may be divided into four groups of 5 years each and used effectively for the determination of four unknown weighting factors, as shown in Table 10, Item 54.

After a searching investigation and numerous tests, the most significant of which are summarized in Tables 10, it was found that the method of least squares (with all the extra labor that the procedure involves, particularly in the formation of normal equations) yields either identical results that accrue from the simplest algebraic approach with simultaneous equations, or a generally comparable series of factors. Occasionally, the results from the least squares procedure show up to advantage, but more often the advantage was with the simpler approach. Wherever the constant value is included among the unknowns, the summations of normal equations direct the solution toward the series of factors that will satisfy the arithmetic means of the observation equation coefficients and, therefore, will not likely satisfy any one of them separately.

The main objective of Tables 10 is to illustrate some of the methods used both in extending the records and also in checking the values thus derived as to their consistency, one with another, in the light of such fragmentary or local records and such other information or authoritative interpretations as were available.

These and associated tests have disclosed errors in some of the published basic data, and have indicated the required correction. For example, the mean annual discharge of the Missouri River at Hermann, Mo., for the calendar year 1925, once published as the equivalent of 76,130 cu ft per sec, was found to be more nearly 50,000 cu ft per sec by various tests similar or related to those in Tables 10, and also by reference to the established rainfall-runoff

TABLE 10.—OUTSTANDING COMBINATIONS OF TRIBUTARIES AND TEST PERI
DETERMINED EITHER BY THE SIMPLEST SOLUTION OF SIMULTANE

"Weighting Factors" Arranged in Columns Headed by the Names of Rivers
and Checking Periods Listed in Co

Item No.	Test period	Gaged area (%)						GA
			Huntington, W. Va.	Ft. Benton, Mont.	Le Claire, Iowa	De Valls Bluff, Ark.	Rich Fountain, Mo.	Garla Ark
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
River			Ohio	Missouri	Mississippi	White	Gasconade	Re
Drainage area, sq miles			55,200[h]	24,600[h]	88,600[h]	23,800[h]	3,180[h]	51,5
								COMBINED
Gaging station			Metropolis,	Hermann,	Keokuk,	Clarendon,	Little Rock,	Alexan
State			Ill.	Mo.	Iowa	Ark.	Ark.	La
River			Ohio	Missouri	Mississippi	White	Arkansas	Re
Drainage area, sq miles			203,000	528,200	119,000	25,750	157,900	65,9
5[b]	1894–1895[g]	11.1	....	....	8.1266	....	....	....
7[b]	1934–1938[g]	15.4	....	−23.7704[h]	3.1095	....	92.9170[h]	....
17[b]	1939–1940[g]	1.0	....	....	....	....	204.3807[h]	....
18[i]	1938–1939[g]	2.2	....	....	....	10.5687	118.7696[h]	....
18[i]	1938–1939[g]	0.4	....	....	....	....	193.8274[h]	....
19	1934–1938	83.2	1.05125	2.76405	0.72409	3.80011	0.68856	....
21	1933–1938[g]	87.2	1.77613	2.04285	−2.34821	−5.23820[h]	0.44926	13.287
22	1935–1940[g]	87.2[f]	1.01424	3.46250	−0.31917	4.86462[h]	−0.39241	3.315
22[i]	1934–1939[g]	87.3[f]	0.843654	4.297565	0.223160	4.475945	−1.78116	3.475
22[i]	1934–1939[g]	88.5[f]	0.878997	4.210438	0.182211	4.03473	−0.87311	1.355
23	1934–1938[g]	85.3[f]	1.527006	−0.540756	0.534084	....	5.18984	0.376
23[i]	1934–1938[g]	73.7[f]	1.287651	4.367897	−1.283654	....	....	3.643
26	1927–1929	74.3	1.582193	0.606386	2.455054	....	....	....
28	1933–1934	28.1	1.637119	....	2.511146	....	....	....
29	1933–1934	63.9	1.545347	2.548392	....	....	....	....
32	1937–1938	28.1	1.422663	....	2.865939	....	....	....
								ITEMS 35 T
45[k]	1933–1939	87.3	1.214501	3.054400	−0.337125	3.110661	−2.672776	8.667
46[k]	1933–1938	90.3	1.163258	2.081342	0.105700	3.111428	0.305337	....
47	1933–1939	87.3	1.107809	2.473716	0.664610	3.224007	0.624616	1.395
48	1933–1938	90.3	1.792211	−5.738581	1.417405	−1.240462	7.911413	....
49	1933–1938	87.3	1.216067	1.495442	0.739574	2.455200	1.342141	2.249
52	1934–1938	83.2	1.051546	2.761618	0.724266	3.799658	0.690306	...
Using 5-yr Means:								
53[i]	1914–1938	75.8	140.680963	65.24110	−371.38345	−472.67235	....	....
53[i]	1914–1938	69.8	174.631612	80.35262	−458.24473	−586.23519	....	....
54	1919–1938	69.8	2.523752	0.271677	−3.146566	3.054882	....	....
57	1919–1938	73.6	2.622843	1.016706	−6.312512	....	....	....
58	1924–1938	67.8	2.944415	0.910599	−4.210654	....	....	....
60	1924–1938	28.0	2.344919	....	−2.694294	4.353857	....	....
64	1929–1938	25.9	3.789960	....	−7.507301	....	....	....
Yearly Means:								
66	1934–1938	69.8	0.984279	3.492675	0.622489	4.178986	....	....
67	1934–1938	75.8	1.021313	2.637703	0.253073	3.106944	....	....
								ITEMS 68 T
115[l]	1932–1941[g]	90.2	1.233898	1.233898	1.233898	1.233898[h]	1.233898	..
116[l]	1932–1941[g]	83.0	1.375846	1.375846	1.375846	1.375846[h]	1.375846	..

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[a] Lower river stations. [b] For additional values in Cols. 8 to 17 inclusive, see Tables 10*A* and 10*B*. [e] W are drought years except 1939 (an average year). [f] With Atchafalaya, Simmesport, La. [g] Discharge da [i] Additional related tests. [j] Some revisions of data from *Transactions*, American Geophysical Union, 19 for the test period were divided by the sum of designated tributary discharges to obtain the ratios, or un

Extending River Discharge Data by the Use of "Weighting Factors,"
ebraic Equations or by Application of Least Squares Procedure
ing Stations to which They Are Related, for Test Periods Listed in Col. 2
Cols. 11 to 17, See Table 10A)

ions:	Checking Period		Departures of Derived Discharges from Observed or Other Basic Data (%)										Item No.
			Checking Period			Selected Years Showing Wide Range of Yield							
sburg, Miss.	No. years	Range of years	Mean	Absolute^{d}	Maximum	1927^{e}	1931	1932	1937^{e}	1939	1940	1941	
3)^{a,b,c}	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(1)
sissippi 44,500^{h}	....	....	....	....	....	....	....	....	....	....	....	....	....
ng Stations at:													
gola^{f} La. sissippi 42,700	....	....	....	....	....	....	....	....	....	....	....	....	....
000^{h}	20	1886–1905^{g}	−15.8	±16.7	−39.6	−19.5	+ 8.1	− 6.8	−17.8	+ 1.9	+19.9	+58.8	5^{b}
000^{h}	65	1875–1939^{g}	+2.2	9.6	−23.9	− 6.7	+14.6	−19.4	0.0	+ 0.6	−44.3	+16.5	7^{b}
000^{h}	20	1922–1941^{g}	+3.5	21.0	+46.3	+40.5	+46.3	−44.9	− 4.7	0.0	0.0	+18.5	17^{b}
...	20	1921–1940^{g}	−0.7	13.5	−41.1	+16.8	+21.0	−33.0	−12.6	0.0	+ 0.5	− 0.8	18^{i}
...	20	1921–1940^{g}	+2.3	15.5	+50.7	+18.7	+29.3	−37.9	− 3.8	0.0	+ 6.3	+12.5	18^{i}
...	50	1889–1938	+3.9	5.1	+19.0	+ 8.7	+ 6.5	− 4.6	+ 0.0	− 1.4	− 7.6	....	19
...	15	1927–1941^{g}	−3.2	7.0	−22.3	−18.6	− 8.2	+ 7.7	0.0	−22.3	+ 4.4	+18.1	21
...	22	1920–1941^{g}	+2.3	3.6	+10.2	+ 2.3	+ 8.5	− 1.2	0.0	0.0	0.0	+ 5.6	22
...	15	1927–1941^{g}	+1.2	2.6	+12.3	− 1.7	+ 8.9	− 2.8	0.0	0.0	− 0.5	+ 3.8	22^{i}
...	15	1927–1941^{g}	+2.3	2.8	+11.8	+ 4.6	+ 9.0	− 1.5	0.0	0.0	− 1.7	+ 4.9	22^{i}
...	15	1927–1941^{g}	−3.9	4.7	−15.1	− 5.6	−15.1	− 1.7	0.0	−10.2	−10.1	+ 1.6	23
25,037	15	1927–1941^{g}	+1.5	2.7	+11.6	+ 8.4	− 1.2	+ 0.6	0.0	− 7.3	− 0.8	+11.6	23^{i}
000^{h}	22 52	1919–1940 1889–1940	+5.7 +6.7	7.9 9.2	+23.6 +33.7	0.0	− 3.5	+ 4.4	+10.9	+ 9.7	+18.7	....	26
000^{h}	22	1919–1940	+1.2	7.1	+21.4	− 6.3	−12.9	+ 0.5	+ 8.7	+ 7.5	+16.9	....	28
000^{h}	22	1919–1940	+3.1	7.3	+27.9	+ 9.2	− 4.6	+ 5.8	+ 6.8	+ 2.8	+ 8.2	....	29
000^{h}	22 52	1919–1940 1889–1940	−6.0 −4.2	8.2 7.9	−19.2 +20.4	−13.4	−17.5	− 7.3	0.0	− 0.2	+ 8.8	....	32
Table 10*A*													
17,176	52	1889–1940	+1.8	8.3	−29.2	− 9.9	− 5.2	+ 4.3	+ 2.1	− 0.6	+ 7.7	....	45^{k}
11,060^{h}	52	1889–1940	+3.5	5.3	−29.4	+15.5	+ 1.2	− 0.8	+ 2.0	+ 3.2	+ 5.2	....	46^{k}
-2,181	52	1889–1940	+3.0	3.5	+13.7	+ 4.5	+ 3.2	− 2.3	0.0	0.0	− 2.6	....	47
55,066^{h}	52	1889–1940	−13.0	15.5	−54.6	−13.8	−29.0	−13.9	0.0	−14.9	+ 2.8	....	48
000	52 10	1889–1940 1923–1932	+0.9 +3.7	5.6 4.0	+20.4 +10.5	− 0.4	− 2.0	− 3.2	0.0	+ 5.6	− 1.9	....	49
000	10	1923–1932	+5.4	9.0	+15.6	+ 8.7	+ 6.8	− 2.8	0.0	+ 5.8	+11.4	....	52
69,256^{h}	10	1923–1932	−162.4			(Too many conditions imposed)							53^{i}
12.098	10	1923–1932	−179.8			(Too many conditions imposed)							53^{i}
000	10	1923–1932	−0.7	6.0	−20.2	− 2.9	−20.2	+ 2.5	+18.8	+10.0	+ 5.6	....	54
17,921^{h}	10	1923–1932	−0.5	12.0	−23.6	− 0.6	−12.1	+12.3	+21.9	+ 6.9	+21.0	....	57
000	10	1923–1932	−1.6	10.4	−24.9	− 6.4	−24.9	+ 8.4	+23.1	+ 8.0	+ 6.6	....	58
000	10	1923–1932	−0.3	5.9	−18.2	− 1.5	−18.2	− 0.5	+17.0	+10.8	+ 5.2	....	60
000	10	1923–1932	−3.7	14.4	−37.9	− 7.0	−37.9	+12.9	+34.3	+11.9	+ 9.8	....	64
-3,203	10	1923–1932	+10.8	11.0	+19.3	+10.8	+ 9.5	− 1.8	0.0	+ 3.2	− 3.8	....	66
18,359^{h}	10 52	1923–1932 1889–1940	+8.2 +4.2	8.2 5.7	+16.1 +18.2	+16.1	+ 5.2	− 0.7	0.0	+ 2.5	+ 4.8	....	67
Table 10*B*													
000^{h}	15	1927–1941^{g}	+1.6	2.5	− 5.2	+ 2.6	+ 3.0	+ 0.4	− 2.2	− 3.4	+ 1.0	+ 4.0	115^{l}
000^{h}	15	1927–1941^{g}	−1.2	4.2	+10.0	− 7.2	− 4.0	+ 0.2	− 2.5	− 0.5	− 1.4	− 2.8	116^{l}

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actors constant. [d] Mean absolute (values approximate). [e] Flood year, in the columns, unmarked years years ending September 30 of each year so listed. [h] Referring to the gaging stations in the main headings. [k] Computed by least squares, and normal equations solved. [l] Mean discharges at the lower river station ting factors above listed.

relationships as applied to that year at the stations involved. Likewise, the numerous maximum departure percentages recorded in Col. 25 indicate that the lower station discharges for the year 1931 are probably deficient and should be increased by at least 10%, revising data of that year prior to July, when the present station was established.

TABLE 10*A*.—SUPPLEMENT TO TABLE

Item No.	Test period[g]	Gaged area (%)	GAGING S						
			McConnelsville, Ohio	Alderson, W. Va.	Chattanooga, Tenn.	Nashville, Tenn.	Lockport, Ky.	Hamilton, Ohio	Chillicothe, Ohio
(1)	(2)	(3)	(10)[n]	(11)	(12)	(13)	(14)	(15)	(16)
River			Muskingum	Greenbrier	Tennessee	Cumberland	Kentucky,	Miami	Scioto
Drainage area, sq miles			7,411[h]	1,357[h]	21,400[h]	12,860[h]	6,310[h]	3,639[h]	3,847[h]
			COMBINED WITH GAGE						
Gaging station			Mt. Carmel,	Kanawha Falls,	Johnsonville,	Clarksville,	Avonmore,	Fetterman,	Red Hou
State			Ill.	W. Va.	Tenn.	Tenn.	Pa.	W. Va.	N. Y.
River			Wabash	Kanawha	Tennessee	Cumberland	Kiskiminetas	Tygart	Alleghen
Drainage area, sq miles			28,600	8,367	38,520	16,000	1,723	1,304	1,690
5[n]	1894–1895	11.1	....	10.5935	....	....	....	....	....
7[n]	1934–1938	15.4	....	−26.6305	16.3435[h]	....	....	....	....
17[n]	1939–1940	1.0	....	0.2964	....	....	....	....	....
18[i,n]	1938–1939	0.4	....	47.4144[h]	....	....	....	....	....
			ITEMS 19 TO						
35	1935–1939	18.3	15.4156[h]	....	....	0.87738[h]	13.0246[h]	21.4527[h]	−20.0958
36[q]	1935–1937	14.6	14.60[o]	....	....	3.32[o]	7.13[o]	....	....
40	1935–1939	38.7	2.88126	18.1615	5.00196[o]	....	....	−166.1619	40.7842
41	1935–1937	37.2	4.96272	7.3593	0.85418[o]	....	....	....	....
44[i]	1937–1938	28.6	4.8195[h]	3.3603	....	....	....	....	....
44[i]	1937–1939	26.7	1.2877[h]	23.1711[h]	....	....	....	....	2.4188
44[i]	1938–1939	15.9	5.5771[h]	18.3464[h]	....	....	....	....	....
44[l]	1937–1938	5.6	....	33.5852[h]	....	....	5.79404	....	....
44[i]	1937–1938	1.5	....	75.7273[h]	....	....	41.13051	....	....
			ITEMS 45 TO 1						
112[l]	1932–1941	18.3	5.437880[h]	....	....	5.437880	5.437880[h]	5.437880[h]	5.4378
113[l]	1932–1941	38.7	2.453260	2.453260	2.453260	....	....	2.453260	2.4532
114[l]	1932–1941	57.0	1.690571	1.690571	1.690571	1.690571	1.690571	1.690571	1.6905

[a] Lower river stations. [c] Weighting factors constant. [d] Mean absolute (values approximate). [e] Fl water years ending September 30 of each year so listed. [h] Referring to the gaging stations in the main headin sums of designated tributary discharges to obtain the ratios, or uniform weighting factors listed above. [m] In It [n] For additional values in Cols. 4 to 9, see Tables 10 and 10*B*. [o] Including the latest revisions of both obser

The method of least squares brings identical results with those from the simplest algebraic method whenever the unknowns, the tributary stations, and years or other time units in the test period are all equal in number. Values derived in connection with a constant term, which automatically extends the test period by one year, represent mean discharges for the entire test period, and therefore should not normally satisfy any of the individual observation equations.

The aforementioned observations are not intended to disparage the use of the least squares procedure in its proper field, for it has done much to facilitate scientific analysis and research. However, the criteria for the application of that method are violated mostly in the hydrologic field, especially as to a large number of observations made with equal care and skill, all yielding small deviations from the mean. Items 45 and 46 in Table 10 represent the best of

EMS 35 TO 44 AND ITEMS 112 TO 114)

IS:	CHECKING PERIOD		DEPARTURES OF DERIVED DISCHARGES FROM OBSERVED OR OTHER BASIC DATA (%)										Item No.
			Checking Period			Selected Years, Showing Wide Range of Yield							
ınting-ton,[a] . Va. (17)[c]	No. years (19)	Range of years[g] (20)	Mean (21)	Absolute[d] (22)	Maximum (23)	1927[e] (24)	1931 (25)	1932 (26)	1937[e] (27)	1939 (28)	1940 (29)	1941 (30)	(1)
Ohio	....	....	....	....	....	....	....	....	....	....	....	....	....
5,200[h]	....	....	....	....	....	....	....	....	....	....	....	....	....
TIONS AT:													
tropolis,	....	....	....	....	....	....	....	....	....	....	....	....	....
Ill.	....	....	....	....	....	....	....	....	....	....	....	....	....
Ohio	....	....	....	....	....	....	....	....	....	....	....	....	....
3,000	....	....	....	....	....	....	....	....	....	....	....	....	....
....	20	1886–1905	−15.8	±16.7	−39.6	−19.5	+ 8.1	− 6.8	−17.8	+ 1.9	+19.9	+58.8	5[n]
....	65	1875–1939	+ 2.2	9.6	−23.9	− 6.7	+14.6	−19.4	0.0	+ 0.6	−44.3	+16.5	7[n]
....	20	1922–1941	+ 3.5	21.0	+46.3	+40.5	+46.3	−44.9	− 4.7	0.0	0.0	+18.5	17[n]
....	20	1921–1940	+ 2.3	15.5	+50.7	+18.7	+29.3	−37.9	− 3.8	0.0	+ 6.3	+12.5	18[i,n]
TABLE 10													
000[o]	15	1927–1941	− 6.1	6.6	−21.1	−11.9	−21.1	− 8.4	0.0	0.0	+ 3.8	− 1.9	35
000	15	1927–1941	− 2.8	4.8	−12.3	− 5.5	−12.3	+ 1.6	− 0.1	− 2.4	+10.5	+ 5.7	36[q]
000[o]	{15 43	1927–1941 1899–1941	+12.8 +10.7	17.6	+57.5	− 7.1	+26.7	+35.3	0.0	0.0	−11.5	−29.8	40
000	15	1927–1941	+ 6.0	7.4	+20.7	+11.1	+14.0	+ 8.1	+ 0.3	− 2.8	− 5.7	− 8.9	41
000[h]	10	1931–1940	− 0.5	4.2	−12.7	− 6.1	+ 0.2	+ 0.2	0.0	− 8.4	+ 3.2	− 0.1	44[i]
000[h]	10	1931–1940	+ 9.5	10.1	+19.1	+11.1	+17.2	+19.1	0.0	0.0	+11.9	+ 5.5	44[i]
000[h]	10	1931–1940	+ 5.6	8.6	+16.9	+12.0	− 7.9	+11.0	+ 8.9	0.0	+ 9.6	− 0.4	44[i]
000[h]	15	1927–1941	+14.2	14.9	+43.8	+16.4	+28.6	+26.3	0.0	+ 3.0	+11.8	− 0.4	44[l]
000	15	1927–1941	+ 3.6	19.0	+44.6	− 9.7	+44.6	+ 2.1	0.0	−18.3	+40.6	+35.5	44[i]
TABLES 10 AND 10*B*													
000	15	1927–1941	+ 0.2	4.2	−14.0	+ 1.4	−14.0	+11.5	+ 0.1	− 1.2	+ 1.5	− 6.9	112[l]
000	15	1927–1941	+ 1.8	7.4	+14.0	− 1.5	+12.6	+12.0	− 9.3	− 6.1	−11.9	+ 0.5	113[l]
...[p]	15	1927–1941	+ 1.3	5.0	+11.8	− 0.6	+ 4.3	+11.8	− 6.4	− 4.6	− 7.7	− 1.8	114[l]

in these columns unmarked years are drought years except 1939 (an average year). [g] Discharge data for
ditional related tests. [l] Mean discharges at the lower river station for the test period were divided by the
ol. 16, the assumption indicated refers to the gaging station at Franklin, Pa. (drainage area, 5,982 sq miles).
derived in estimated data. [p] All ten stations. [q] Slide-rule calculations only.

TABLE 10*B*.—Supplement to Table

Item No. (1)	Test period[g] (2)	Gaged area % (3)	Gaging Stations: Rich Fountain, Mo. (8)[q]	Alderson, W. Va. (11)[q]	Vicksburg, Miss. (18)[a,e,r]	Checking Perio No. years (19)	Range o years[g] (20)
River			Gasconade	Greenbrier	Mississippi	....	....
Drainage area, sq miles			3,180[h]	1,357[h]	1,144,500[h]	....	....
Combined with Ga							
Gaging station			Little Rock, Ark.	Kanawha Falls, W. Va.	Angola,[f] La.	....	....
State						....	....
River			Arkansas	Kanawha	Mississippi	....	....
Drainage area, sq miles			157,900	8,367	1,242,700	....	....
Items 5 to 67,							
1922–1941 Means, and:							
68	1940–1941	1.0	181.34312[h]	14.29019	−115,981[h]	20	1922–19
69	1940–1941	0.9	201.82239[h]	16.82064	−129,155	20	1922–19
Yearly Means Only:							
75	1938–1939	0.9	203.08791[h]	5.351215	000	20	1922–19
1922–1941 Means, and:							
78	1940–1941	0.4	182.89615[h]	65.182267[h]	−88,208[h]	20	1922–19
83	1940	0.4	207.26092[h]	23.837951[h]	000	20	1922–19
91	1939–1940	0.4	204.957790[h]	25.756673[h]	000	20	1922–19
1922–1941 Means, and:							
92	1941	0.4	85.889908[h]	152.810789[h]	000[h]	20	1922–19
93	1941	0.4	97.566239[h]	172.553078[h]	000	20	1922–19
Five-Year Means Only:							
94	1927–1941	1.0	97.071751[h]	25.091831	−13,417[h]	20	1922–19
95	1927–1941	0.9	117.817958[h]	22.326052	24,910	20 53	1922–19 1889–19
96	1932–1941	1.0	93.211972[h]	24.712427	000[h]	20	1922–19
97	1932–1941	0.9	124.986265[h]	23.030062	000	20	1922–19
98	1927–1941	0.4	106.886839[h]	96.660423[h]	52,504[h]	20 53	1922–19 1889–19
99	1927–1941	0.4	126.551046[h]	86.006116[h]	83,564	20 53	1922–19 1889–19
100	1932–1941	0.4	123.831488[h]	102.838480[h]	000[h]	20	1922–19
101	1932–1941	0.4	153.519985[h]	95.839137[h]	000	20 53	1922–19 1889–19
One-Year Means:[k]							
102	1927–1941	0.4	85.129169[h]	116.394081[h]	74,377[h]	20	1922–19
103	1927–1941	0.4	108.391795[h]	117.171683[h]	72,306	20	1922–19
104	1927–1941	1.0	84.774019[h]	21.541561	62,064[h]	20	1922–19
105	1927–1941	0.9	108.819457[h]	21.004572	65,498	20	1922–19
106	1932–1941	1.0	78.531555[h]	32.185600	−60,249[h]	20	1922–19
107	1932–1941	0.9	92.958831[h]	34.626670	−67,045	20	1922–19
Ten-Year Means Only:[l,r]							
108	1932–1941	1.0	36.442748[h]	36.442748	000[h]	20	1922–19
109	1932–1941	0.9	40.635115[h]	40.635115	000	20	1922–19
110	1932–1941	0.4	114.080564[h]	114.080564[h]	000[h]	20	1922–19
111	1932–1941	0.4	127.204370[h]	127.204370[h]	000	20 53	1922–19 1889–19
113	1932–1941	38.7	....	2.453260	....	15	1927–19
114	1932–1941	57.0	....	1.690571	....	15	1927–19
115	1932–1941	90.2	1.233898	....	000[h]	15	1927–19
116	1932–1941	83.0	1.375846	....	000[h]	15	1927–19

[a] Lower river stations. [c] Weighting factors constant. [d] Mean absolute (values approximate). [e]] Simmesport, La. [g] Discharge data for water years ending September 30 of each year so listed. [h] Refe [l] Mean discharges at the lower river station for the test period were divided by the sums of designated trib missing columns, see Tables 10 and 10*A*.

EMS 68 TO 111)

Departures of Derived Discharges from Observed and Other Basic Data (%)										Item No.
Checking Period			Selected Years, Showing Wide Range of Yield							
ean 1)	Absolute[d] (22)	Maximum (23)	1927[e] (24)	1931 (25)	1932 (26)	1937[e] (27)	1939 (28)	1940 (29)	1941 (30)	(1)
..	....	....	....	....	....	....	....	....	....	....
..	....	....	....	....	....	....	....	....	....	....
TIONS AT:										
..	....	....	....	....	....	....	....	....	....	....
..	....	....	....	....	....	....	....	....	....	....
..	....	....	....	....	....	....	....	....	....	....
..	....	....	....	....	....	....	....	....	....	....
LES 10 AND 10*A*										
.0	18.2	−44.6	+34.5	+27.8	−44.6	− 8.4	− 6.8	0.0	0.0	68
.0	15.1	−43.3	+22.6	+16.1	−43.3	− 5.3	− 2.7	0.0	0.0	69
.0	15.1	+42.5	+19.7	+28.7	−42.3	− 5.1	0.0	+ 1.5	+13.6	75
.0	18.0	−42.3	+34.7	+28.8	−42.3	− 7.9	− 4.8	0.0	0.0	78
.0	15.7	±42.5	+20.7	+27.6	−42.5	− 5.1	+ 0.4	0.0	+12.5	83
.7	15.1	−42.4	+19.9	+27.1	−42.4	− 5.5	0.0	0.0	+ 6.1	91
.0	12.9	+36.7	+ 2.4	+36.7	−20.3	−10.7	− 7.6	+30.5	0.0	92
.0	12.4	+29.2	− 6.6	+24.0	−19.2	− 7.5	− 3.4	+29.2	0.0	93
.8 .3	13.5	+38.9	+ 4.4	+38.9	−28.7	−12.2	−10.9	+18.4	+ 6.2	94
.04	11.6	−29.9	− 4.0	+28.5	−29.9	− 9.9	− 6.7	+20.8	+ 3.8	95
.0	13.7	+40.5	+ 2.6	+40.5	−28.1	−12.6	−11.1	+19.3	+ 7.2	96
.8	11.2	−30.9	− 1.2	+26.7	−30.9	− 9.3	− 6.3	+19.3	+ 2.1	97
.6 .0	13.9	+43.8	− 6.3	+43.8	−26.4	−10.9	− 6.8	+16.8	+ 9.6	98
.2 .0	11.5	+31.7	− 2.7	+31.7	−28.1	− 8.9	− 3.4	+19.4	+ 6.4	99
.5	13.3	+38.9	+14.1	+38.9	−28.7	− 9.3	− 5.7	+19.8	+ 5.4	100
.6 .2	11.9	−31.4	+ 7.5	+25.5	−31.4	− 6.5	− 1.7	+13.9	+ 0.7	101
.2	13.8	+46.4	− 0.7	+46.4	−21.0	−11.2	− 7.1	+31.3	+10.3	102
.6	12.2	+30.3	− 6.5	+30.3	−23.8	− 9.0	− 3.8	+24.6	+ 8.8	103
.5	15.5	+48.6	− 1.6	+48.6	−25.6	−12.5	− 9.9	+30.6	+14.5	104
.8	12.6	+32.5	− 7.3	+32.5	−28.0	−10.1	− 6.3	+23.8	+13.2	105
.7	13.9	+29.5	− 2.1	+29.5	−28.3	−15.4	−15.6	+25.8	− 3.5	106
.1	11.4	−29.3	− 9.9	+25.9	−29.3	−13.5	−12.7	+10.9	− 4.6	107
.2	16.0	+37.6	−18.0	+35.3	−20.9	−17.9	−18.0	+37.6	− 0.1	108
.8	14.3	+42.6	−25.8	+20.9	−21.1	−16.2	−15.6	+34.2	− 6.7	109
.8	12.8	+37.6	− 1.2	+37.6	−27.1	−10.2	− 6.7	+21.6	+ 3.5	110
.2 .8	11.7	−27.3	−10.6	+22.9	−27.3	− 8.4	− 4.0	+18.7	− 3.4	111
.8	7.4	+14.0	− 1.5	+12.6	+12.0	− 9.3	− 6.1	−11.9	+ 0.5	113
.3	5.0	+11.8	− 0.6	+ 4.3	+11.8	− 6.4	− 4.6	− 7.7	− 1.8	114
.6	2.5	− 5.2	+ 2.6	+ 3.0	+ 0.4	− 2.2	− 3.4	+ 1.0	+ 4.0	115
.2	4.2	+10.0	− 7.2	− 4.0	+ 0.2	− 2.5	− 0.5	− 1.4	− 2.8	116

; in these columns, unmarked years are drought years except 1939 (an average year). [f] With Atchafalaya, ne gaging stations in the main headings. [k] Computed by least squares, and normal equations solved. narges to obtain the ratios, or uniform weighting factors above listed. [r] For further information regarding the least squares tests, with maximum departures of some 29% in a 52-yr checking period, in contrast with 13.7% for Item 47.

According to authoritative analyses of probabilities,[49] the standard error of the standard deviation should be as large as the deviation itself whenever the number, n, of observation equations and unknowns is equal to 3; and for n equals 10, the odds are 3 to 2 that the calculated probable error is within 20% of the correct value; furthermore, the odds are 30 to 1 that the error is within 50%.

The investigations underlying this paper have demonstrated that the suitability or representative character of the observation equations far outweighs the influence of number. Some arrays of data covering only 2 or 3 years have outdone other arrays of 6 or 7 years, or such numbers as 10, 15, or 20. One test of 25 years in groups of five each, yielding a set of five 5-yr means, was found to have imposed so many conditions that the weighting factors were unwieldy and thus useless for the purpose intended, with constants exceeding 6,400,000 cu ft per sec and 8,200,000 cu ft per sec, respectively, for Vicksburg and Angola-Simmesport, Items 53, Table 10.

The only basis thus far disclosed for satisfying the fundamental criterion (that the sum of the squares of deviations shall be a minimum) involves, either directly or indirectly, some multiple of the arithmetic means of the respective coefficients appearing in the observation equations. Comparable results may be had by including the means of all or of a part of the test period, along with any desired number of seemingly representative yearly data, as illustrated by Items 92 and 93, Table 10*B*. Somewhere within the twilight zones of mathematical probability, fantastic legends have grown up, attributing almost magic powers to the least squares procedure, when simpler methods achieve either identical or comparable results, as shown by many items associated with those in Tables 10.

Mr. Cory's comment regarding an extremely low stage of the Ohio River at Cincinnati calls forth a simple test, based on official records as shown in Table 11.[50]

TABLE 11.—RAINFALL DEPTHS IN INCHES AT MARIETTA, OHIO, FOR CRITICAL MONTHS, 1933–1936

Month	1933	1934	1935	1936	Means, 1817–1930
July	6.84	5.08	2.58	3.16	4.63
August	0.25	1.33	6.54	3.16	3.85
September	3.30	0.67	2.75	2.08	3.03
October	3.17	3.75	4.80	2.50	2.91
Yearly depth	40.37	34.66	42.46	36.76	42.25

A casual examination of the foregoing data in the light of the close correlation of Marietta rainfall with runoff as far down the river system as Vicksburg justifies the conclusion that the time of minimum discharge of the Ohio River at Cincinnati was September, 1834, in the second successive month of subnormal rainfall, and in a year of drought following a slightly sub-normal year. As to the actual discharge, it might have been anything from 3,000 to 5,000 cu ft per sec inasmuch as such minima have been observed from Huntington to Louisville, Ky., within recent years. A channel cross section of about 70 ft by 10 ft might have been spanned by those early steamboats. As mean velocities of 4 to 7 ft per sec would account for such minimum discharges, there is no occasion to challenge the veteran's boyhood memories; they correspond with the probable conditions of flow.

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[49] "Handbook of Engineering Fundamentals," by Ovid W. Eschbach, Am. Telephone and Telegraph Co., John Wiley & Sons, Inc., New York, N. Y., 1936, Chapter 2, p. 127.

[50] *Bulletin W*, U. S. Weather Bureau, 1930, and later data.

It is worthy of note that the same year that marked the establishment of methodical rainfall records at Marietta, Ohio, and of Mississippi River gage heights at Natchez, saw the election of the eminent scientist, Pierre Laplace, to the presidency of the French Academy. Napoleon's complaint upon the occasion of his dismissing Laplace from the office of Minister of the Interior had to do with "administration on the basis of differential calculus," or "of infinitesimals," according to other translations.

The accustomed praise accorded the Humphreys and Abbot hydrologic data and findings, coupled with the practical disregard and discounting to which they have been subjected entirely too long, may well have its origin in the application of calculus or the Laplace and Gaussian theories of probabilities to data in the rough, where practical application of experience and the simplest mathematical approach would have served better. Intensive studies of those early data pertaining to the Mississippi River system would have been richly repaid if no other result had been achieved than disclosing the fruitless hours devoted to building up the usual normal equations, preparatory to their solution as simultaneous equations, when the basic observation equations are capable of yielding either identical, fairly comparable, or more satisfactory values of the respective unknowns, as described herein. This comment in no wise circumscribes the use of calculus, probabilities, least squares, or other preferred methods within their proper sphere, but is aimed against ill-advised or improper application of such procedure, either inside or outside their respective provinces. The gross errors inherent in either the instrumentation, the sampling, the observations in the field, or their interpretations in the office, or the summation of these defects, would seem to disqualify the ordinary run-of-the-mill hydrologic data for analysis by the method of least squares, where large numbers of small deviations constitute the most suitable arrays. Nevertheless, the normal alternation of plus and minus errors seems to effect such compensations so as to render the best attainable values of rainfall and runoff; and these have consistently withstood the most practical tests as to their suitability and reliability for the purposes intended—evaluation, conservation, and utilization of water resources.

Corrections for *Transactions:* In *Proceedings*, March, 1942, page 422, Table 1, Footnote [b], change "billions of cubic feet per second" to "billions of cubic feet per year"; on page 432, Table 5, the runoff excess for 1855 should be "– 2.49" instead of "2.49"; and on p. 438, Fig. 3, subcaption (*b*), change "1910" to "1851."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STRAINS, STRESSES, AND SHEAR IN ENGINEERING PROBLEMS

Discussion

BY MESSRS. C. M. GOODRICH, MAURICE P. VAN BUREN, JOSEPH A. WISE, AND DAVID B. HALL

C. M. GOODRICH,[4] **M. AM. SOC. C. E.**[4a]—The newer theories of Sandel and Schleicher[5] are in better agreement with tests than were the older theories. Mr. Sandel considers the change of length in the long diagonal in a cubic element to be the measure of allowable distortion and equates this to the distortion under a single axial load to obtain his criterion of safety. Mr. Schleicher[6] follows the same argument, "Gestaltänderung" (or "Change of Shape"). Their results differ (but not greatly; see Table 2) possibly because of a difference in the value of Poisson's ratio.

TABLE 2.—COMPARISON OF STRESS THEORIES[a]

For:	Sandel	Schleicher
σ_x, σ_y, τ	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - 0.865\ \sigma_x\ \sigma_y + 2.865\ \tau^2}$	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - \sigma_x\ \sigma_y + 3\ \tau^2}$
σ_x, σ_y	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - 0.865\ \sigma_x\ \sigma_y}$	$\sigma = \sqrt{\sigma^2_x + \sigma^2_y - \sigma_x\ \sigma_y}$
σ_x, τ	$\sigma = \sqrt{\sigma^2_x + 2.865\ \tau^2}$	$\sigma = \sqrt{\sigma^2_x + 3\ \tau^2}$
τ	$\sigma = 1.69\ \tau$	$\sigma = \sqrt{3}\ \tau$

[a] σ_x is the unit stress in one direction, σ_y is the unit stress at right angles to σ_x, and τ is the shear.

To illustrate the procedure, consider the case of pure shear in comparison with tension. Under a single tensile load the distortion $\epsilon_1 = \frac{\sigma}{E}$ and $\epsilon_2 = \epsilon_3 = -\frac{\mu\ \sigma}{E}$; and for the diagonal, $\epsilon = \sqrt{\epsilon^2_1 + \epsilon^2_2 + \epsilon^2_3} = \frac{1}{E}\sqrt{\sigma^2 + 2\ \mu^2\ \sigma^2}$. If

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NOTE.—This paper by Silas H. Woodard, M. Am. Soc. C. E., was published in February, 1943, *Proceedings*.

[4] Cons. Engr., The Canadian Bridge Co., Ltd., Walkerville, Ont., Canada.

[4a] Received by the Secretary February 23, 1943.

[5] *Stahlbaukalender*, 1937, p. 24.

[6] *Bauingenieur*, 1928, p. 253.

$\mu = 0.3,$

$$\epsilon = 1.086 \frac{\sigma}{E} \qquad (22)$$

In a case of shear only (see Fig. 6), since the cube of investigation is infinitely small, a cube of the same size at 45° with the old cube is its equivalent. In this new cube each unit axial stress equals the old unit shear stress. The change of length of the diagonal in the square is $\epsilon = \sqrt{\epsilon^2{}_1 + \epsilon^2{}_2}$. Now $\epsilon_1 = \frac{\tau}{E}(1 + \mu)$, and $\epsilon_2 = -\frac{\tau}{E}(1 + \mu)$, whereas $\epsilon_3 = 0$.

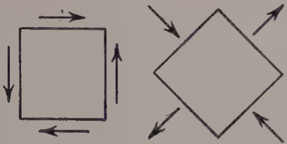

Fig. 6

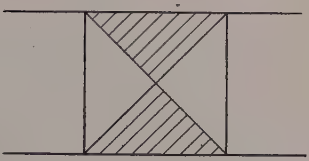

Fig. 7

Accordingly, the distortion of the long diagonal of the square (which, since $\epsilon_3 = 0$, is also the distortion of the long diagonal of the cube) is

$$\epsilon = \frac{\tau}{E} \sqrt{2\,(1 + 2\,\mu + \mu^2)}; \text{ or, if } \mu = 0.3:$$

$$\epsilon = 1.8386 \frac{\tau}{E} \qquad (23)$$

Equating formulas 22 and 23

$$\sigma = 1.69\,\tau \qquad (24)$$

When the bearing surfaces testing a stone cube are well greased,[7] the characteristic pyramids shown in Fig. 7 no longer appear, and the pressure required to break the cube drops to one half or less of what it is where friction helps the specimen.

The stresses indicated in Table 2 are to be taken as equivalent stresses. Consider a steel cube under compression in the vertical direction; its sides bulge and then flow when the elastic limit is exceeded. The writer was taught, and still believes, that the sum of all forces in any direction should equal zero.

Maurice P. van Buren,[8] Assoc. M. Am. Soc. C. E.[8a]—Interesting adventures in mathematics can be developed from false premises. In referring to a concrete cylinder subjected to axial pressure in a compression testing machine (see heading, "Fundamental Problems of Stress in Terms of Strain"), the author states

> "* * * By theory the diameter of the cylinder will expand in the proportion of Poisson's ratio and, acting normal to the direction of the compressive stress, there will be a tensile stress of μ times the compressive stress."

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[7] "Vorlesungen über Technische Mechanik," by August Föppl, 9th Ed., III 68; see also "Zwang und Drang."

[8] Cons. Engr. (J. Di Stasio & Co.), New York, N. Y.

[8a] Received by the Secretary February 23, 1943.

From this postulate, various unusual conclusions are derived. One of these is Eq. 12 which expresses a difference between the intensity of normal force and normal stress on a plane.

In the generally accepted theory, the effect of an axial force on the dimensions of a body resembles temperature change in that, for given conditions, there is a natural size and shape which the body tends to assume. Lateral expansion or contraction from such causes produces stress only if prevented, in which case the stress will be of the same kind as the axial force. This is obvious from Fig. 1, where, by the principles of equilibrium, there can be no resultant lateral force unless external compression forces are applied to the sides of the piece. However, in short thick cylinders, the inner layers may be restrained by the resistance of less heavily loaded outer fibers, causing a balance of equal and opposite internal circumferential forces. The amount of these forces will bear a definite relation to Poisson's ratio and may explain lateral tension failures under compressive tests of certain brittle materials already subject to high initial cohesive stresses.

If derived from correct concepts, formulas expressing the stresses and strains involved in lateral expansion and contraction will follow accepted theory and can be found in any standard handbook of mechanics. Such deformations naturally affect the behavior of all materials, to a greater or less extent, depending on particular characteristics, shape, loading, and initial stress. In any event, in themselves, the deformations prove nothing regarding the fundamental causes of failure in general.

JOSEPH A. WISE,[9] M. AM. SOC. C. E.[9a]—Some interesting ideas are presented in this paper, but unfortunately some fundamental errors are involved that vitiate most of the conclusions. The first of these errors is the tacit assumption that in an elastic solid there must be a stress proportionate to, and in the direction of, every strain. Consider the illustration, used in the paper, of a concrete cylinder with a uniformly distributed compressive force applied axially. There is a lateral strain of μ times the axial strain (μ being Poisson's ratio), but it does not follow that there is any lateral stress. Incidentally the paper limits its discussion to co-planar states of stress, although, for a thorough presentation, a possible triaxial state of stress must be considered.

Fig. 8 shows a concrete cylinder with a compressive load applied in the direction of the z-axis, so that the unit force or unit stress is σ_z. Assume complete freedom of lateral displacement on the end surfaces in order to avoid unnecessary complexities introduced by lateral restraint. The state of stress will be assumed homogeneous; that is, all elements will have the same stress, σ_z. Take an element at the outer surface, as shown. On inspection $\sigma_x = 0$, since there is no external force applied in the x-direction. Similarly, σ_y must be zero, otherwise one could not satisfy the equation of static equilibrium, $\sum y = 0$. From tests it is known, however, that transverse strains occur; and therefore strains occur, but no stresses, in the x-direction and y-direction. It would do no good to assume any hypothetical arrangement of tensions on the outer surface and compressions in the core, as, when the conditions for static equilibrium have been met, the net total strain in the x-direction and in the y-direction would be zero (due to any such set of forces) and would not account for lateral strain. The same error renders Eqs. 8*a* and 8*b* of the paper incorrect. Then in Eq. 8*a*, f_n is not equal to $E\,\delta_n$, and likewise Eq. 8*b* is incorrect since the stress is not necessarily equal to E times the strain. This error is implicit in the complex definition of stress used in the paper and invalidates most of the subsequent work. In defining stress as normal force per unit area which is accompanied by, and is proportional to, strain in the same direction, an unnecessary and unusual limitation is placed upon the concept of "stress" and, by implication, this leads to the erroneous concept that every strain must have a stress proportional to it and in the same direction.

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[9] Lt. Comdr. (CEC), U. S. N. R., Public Works Office, 3d Naval Dist., New York, N. Y.

[9a] Received by the Secretary March 1, 1943.

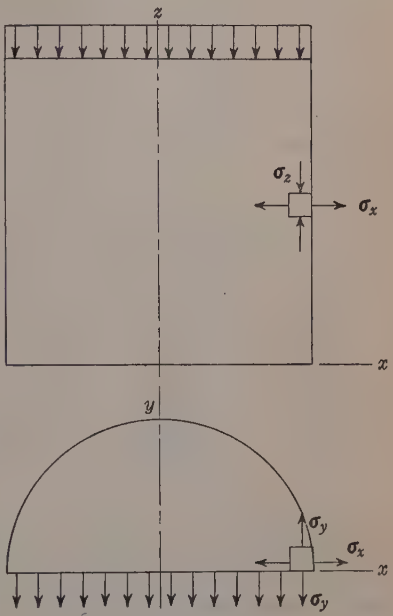

FIG. 8

Definitions are limitations imposed on words so that speech and writing will convey more accurate concepts. In setting up his new definitions of stress, the author does not convey more accurate concepts thereby and no longer is using the language that is basic to the various theories of failure enumerated. The customary definition of stress, heretofore used, is that stress is force per unit area within a solid. This is amplified by defining normal stress (tension or compression) as force per unit area acting perpendicularly to the area in question, whereas shearing stress is force per unit area acting parallel to and on the surface of this area. Since most of the references to the subject under discussion used these definitions, the writer will use them also. It may be pointed out that some of the errors inherent in the paper arise from the definitions which implicitly contain false assumptions. This is the reason for rejecting the proposed definitions and adhering to the customary ones. Based on these generally accepted definitions, the value of the principal stresses for Table 1 of the paper have been recomputed in Table 3.

In stating that the Maximum Stress Theory and the Maximum Strain Theory are inseparable, the author fosters a misconception that stems from the proposed definitions. The Maximum Stress Theory is the naive one which states that materials rupture when a definite maximum normal stress exists. This has been shown to be untenable, and Table 3 shows how far from true it is. The tensile strength of the aluminum was 23,400 lb per sq in.; yet all the specimens failed at lower tensile stresses. In test 3SA7, although the tensile stress was only 56% of the ultimate tensile strength, rupture occurred.

The Maximum Strain Theory, which states that materials fail when a maximum elongational strain occurs, is the theory usually considered the explanation for the failure of brittle materials such as concrete. The interpretation that the strain must be due to a stress in the same direction is incorrect, as has been shown. The proper interpretation of tests on concrete mentioned by the author, according to this theory, is that in pure tension specimens the elongational strain is equal to $\frac{\sigma}{E}$, of course, and failure is ascribed to excessive strain, not excessive stress. In a compression specimen, when the lateral strain reaches this value, failure occurs although there is no lateral stress present.

TABLE 3.—RECOMPUTATION OF MAXIMUM AND MINIMUM STRESSES IN TABLE 1

Test No.	Applied shear	Applied tension	Maximum tension	Maximum compression	Maximum shear
3SA3	10,300	15,700	20,800	5,100	12,950
3SA24	10,300	14,900	20,160	5,262	12,712
3SA15	11,060	13,200	19,433	6,233	12,833
3SA18	11,060	13,800	19,890	6,090	12,990
3SA21	11,060	14,300	20,275	5,975	13,125
3SA4	12,860	11,700	19,978	8,278	14,128
3SA20	12,860	14,100	21,716	7,616	14,666
3SA10	12,720	10,300	18,873	8,573	13,723
3SA17	12,720	9,950	18,632	8,682	13,657
3SA7	13,100	0	13,100	13,100	13,100

The Maximum Shear Theory states that materials fail when the shearing stresses reach the maximum shearing strength of the material. This is in much better agreement with test results on ductile materials than the two preceding theories, as may be seen from Table 3. The shearing strength should be 50% of the ultimate tensile strength, or 11,700 lb per sq in. This theory has been modified by Mohr[10] to make the shearing strength a function of the maximum tensile stress and is in somewhat better agreement with some test results than the unmodified Maximum Shear Theory. Mohr's theory can be best illustrated by means of Mohr's circle of stress as in Fig. 9.

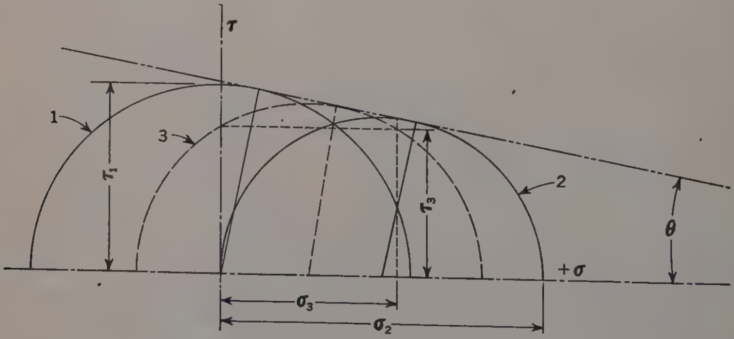

FIG. 9

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[10] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., 1941, pp. 480–482.

Here τ_1 in circle 1 is the maximum shearing strength in pure shear, σ_2 in circle 2 is the maximum tensile strength in pure tension, and σ_3, τ_3 are the tensile and shearing stresses in any combined shear and tension. Mohr's theory states that these circles all touch a common envelope, which approximates a straight line making an angle θ (approximately 15°) with the horizontal. Circles of stress and circles of strain[11] are useful graphical aids in presenting the relationships between stresses and strains.

The Maximum Strain Energy Theory states that materials rupture when the strain energy introduced exceeds a maximum value. The Maximum Distortion Energy Theory states that materials rupture when the distortion energy exceeds a limiting value. Distortion energy is defined as total strain energy minus volume-change energy. This last theory was shown to be in better agreement with test results than the others for the tests referred to in the paper[12] from which Table 1 was taken.

The failure or disruption of materials is a subject that will probably require much further research before engineers can select the best theory of failure, and several theories may be valid, each for its own realm and for certain types of materials. Pending such research, present theories should be regarded as purely tentative and expedient approximations—using the Maximum Strain Theory for brittle materials and either the Mohr Maximum Shear Theory or the Maximum Distortion Energy Theory for ductile materials.

David B. Hall,[13] Assoc. M. Am. Soc. C. E.[13a]—Before discussing any of the conclusions of this paper, the writer finds it necessary to take exception to the terminology used. Throughout the literature of elasticity, plasticity, and strength of materials, the term "stress" is used universally to designate the forces that must act across any plane in a body to maintain equilibrium. Since matter is not continuous but consists of thinly dispersed particles, it is understood, of course, that stress in the foregoing sense is a statistical, or average quantity, much the same as gas pressure, which, although actually consisting of a bombardment by gas molecules, can be, and is, measured in pounds per square inch. There is no reason why the author should abandon the term "stress" and substitute "intensity of force." The interatomic phenomena in a stressed body are no doubt very complicated. The author has elected to reserve the term "stress" for some phenomenon, apparently of this kind, although he has offered no explanation of its nature further than that stress is something that is proportional to strain. As so defined, stress serves no purpose which could not be served equally well by attributing any phenomenon related to strain directly to strain itself. That the author's transposition of accepted terms is confusing is well illustrated by his own statement in his discussion of failure theories: "Although they are inseparable, the Maximum Strain Theory and Maximum Stress Theory are sometimes referred to as two independent theories." The Maximum Stress Theory refers to stress as ordinarily defined and is entirely unrelated to the Maximum Strain Theory. Throughout the remainder of this discussion the writer proposes to use the words "stress" and "strain" in their usual and accepted meanings.

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[11] "Circles of Strain," by Joseph A. Wise, *Journal of the Aeronautical Sciences*, August, 1940, Vol. 7, No. 10, pp. 438–440.

[12] "Failure of Aluminum Subjected to Combined Stresses," by Joseph Marin and R. L. Stanley, *Supplement, The Welding Journal*, February, 1940, Vol. 19, pp. 74s–80s.

[13] Designer, Howard, Needles, Tammen & Bergendoff, New York, N. Y.

[13a] Received by the Secretary March 3, 1943.

The paper presents two conclusions which warrant close examination: (1) There is no such thing as shearing stress or shearing strength; and (2) the failure of materials under combined stress is to be predicted by the Maximum Strain Theory.

1. The assertion that there is no such thing as shearing stress or shearing strength easily could wander off into the fields of metaphysics, but a few definite statements can be made with comparative safety. First, the concept of shear is clear enough: If the resultant stress acting across a plane within a body is inclined to that plane, it can be resolved into a normal component called normal stress and a tangential component called shearing stress. Second, failure may occur along a plane (*a*) if the resistance of the material to rupture is less than the normal stress or (*b*) if the resistance of the material to sliding or plastic flow is less than the shearing stress.

The author's proof of the nonexistence of shear is found to proceed something like this: If shearing stress exists on one plane, then there must be normal stresses on planes inclined at 45° to this plane. Therefore (?) the normal stresses on these inclined planes are real and resist external loads, whereas the shearing stress on the plane first considered is redundant and unnecessary.

The fallacy of this argument is not difficult to discover. The fundamental requirement of equilibrium of stress at a point is that the stresses on the various faces of any small figure surrounding the point shall be in equilibrium. The author has failed to recognize this requirement. In effect, instead of declaring that the stress on one face of a small triangular prism must balance the stresses on the other two (the ends being free of stress), he has stated that stress on one face may be replaced by stresses on the other two.

TABLE 4.—COMPARISON OF FAILURE THEORIES FOR ALUMINUM SHAFTS
(In Pounds per Square Inch)

Test No.	Reported Stress		Principal Stresses			Ratio $\frac{p_0}{23,400}$			
	s	p	p_1	p_2	p_3	Maximum stress	Maximum strain	Maximum shear	Octahedral shear
3SA3	10,300	15.700	20,800	0	−5,100	0.89	0.96	1.11	1.01
3SA24	10,300	14,900	20,150	0	−5,250	0.86	0.94	1.08	0.99
3SA15	11,060	13,200	19,500	0	−6,300	0.83	0.92	1.10	1.00
3SA18	11,060	13,800	19,900	0	−6,100	0.85	0.93	1.11	1.10
3SA21	11,060	14,300	20,300	0	−6,000	0,87	0.95	1.12	1.02
3SA4	12,860	11,700	19,950	0	−8,250	0.85	0.97	1.20	1.07
3SA20	12,860	14,100	21,700	0	−7,600	0.93	1.03	1.25	1.13
3SA10	12,720	10,300	18,850	0	−8,550	0.81	0.93	1.17	1.04
3SA17	12,720	9,950	18,600	0	−8,700	0.79	0.92	1.17	1.03
3SA7	13,100	0	13,100	0	−13,100	0.56	0.75	1.12	0.97

2. The failure of materials under combined stress is to be predicted by the Maximum Strain Theory. Table 4 has been prepared to compare the failure

stresses reported in Table 1, for some aluminum shafts, with the failure stresses predicted by four common theories. This has been done by calculating a simple axial tension assumed to be equivalent to the combined stress reported and by comparing it with the known tensile strength of 23,400 lb per sq in. The three principal stresses were first obtained:

$$p_2 = 0 \quad \text{(25a)}$$

$$\left.\begin{matrix} p_1 \\ p_3 \end{matrix}\right\} = \frac{1}{2}p \pm \sqrt{\left(\frac{p}{2}\right)^2 + s^2} \quad \text{(25b)}$$

in which p_1, p_2, p_3 = principal stresses, in descending order of magnitude, p = reported tensile stress, and s = reported shear stress. The equivalent simple tension, p_0, was then obtained in terms of p_1, p_2, and p_3, according to the rules for each of the four theories: For Maximum Stress Theory—

$$p_0 = p_1 \quad \text{(26a)}$$

for Maximum Strain Theory—

$$p_0 = p_1 - \mu\,(p_2 + p_3) \quad \text{(26b)}$$

for Maximum Shear Theory—

$$p_0 = (p_1 - p_3) \quad \text{(26c)}$$

and, for Octahedral Shear Theory—

$$p_0 = \sqrt{\frac{(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_1 - p_3)^2}{2}} \quad \text{(26d)}$$

As has been observed generally for ductile materials, the octahedral shear formula gave conspicuously more accurate results than any of the others. Also, it will be noted that the Maximum Strain Theory erred consistently on the side of danger.

Tests on Concrete Cylinders.—In further support of the Maximum Strain Theory the paper cites compression tests performed on concrete cylinders and prisms with lubricated ends. Significant results of the tests were: The specimens failed by splitting; failure occurred under a stress producing lateral extension approximately equal to the elongation at which specimens fail in tension; and failure occurred at substantially lower stresses than in specimens tested in the ordinary manner. The explanation offered by the author to account for the difference between the behavior of lubricated and unlubricated specimens is that the latter are prevented, by end friction, from expanding laterally and thereby are prevented from failing. The answer may not be as simple as this. In particular, the lateral restraint at the ends is a more or less local effect. Therefore, according to the author's hypothesis, (*a*) failure of unlubricated specimens should originate at the middle, where the strengthening effect is least, instead of at the ends as was observed, and (*b*) the strength of restrained specimens should approach that of lubricated specimens as the length is increased.

A condensed summary of the results of the tests is given in Table 5. The anticipated convergence of strength values for increase of length seems to be indicated by the table, although the results are erratic, and the mortar cylinders vary in exactly the opposite manner. Table 5 also shows so great a difference between strengths of plain and lubricated specimens as to suggest that some other factor may have been at work to reduce the strength of the lubricated specimens. The specimens were lubricated by sheets of blotting paper saturated with grease. Conceivably this method may have had two possible effects: There may have been a hydraulic pressure gradient across the ends, resulting in a maximum pressure exceeding the average; and the lubricant may have hastened failure by transmitting hydraulic pressure to all cracks as fast as they formed. These explanations are only speculative, but they do serve to emphasize the precariousness of a combined stress theory based on a comparison of greased and dry concrete columns, instead of on tests of the kind exemplified by Tables 1 and 4.

TABLE 5.—EFFECT OF LENGTH AND END CONDITIONS ON STRENGTH OF MORTAR COMPRESSION SPECIMENS[2]

Length diameter	COMPRESSION STRENGTH FOR PLAIN ENDS / COMPRESSION STRENGTH FOR GREASED ENDS				
	1 : 3 Mortar		Neat Cement		Average all specimens
	2-in. diameter cylinder	4-in. by 4-in. prism	2-in. diameter cylinder	4-in. by 4-in. prism	
1	1.11	1.55	1.83	1.95	1.61
1.5	1.20	1.75	1.80	1.56	1.58
2	....	1.40	1.78	1.28	1.49
3	1.44	....	....	....	1.44
4	1.40	1.05	1.20	1.12	1.19

The evidence presented in support of the maximum strain failure theory thus sums up to two items:

I. Combined shear and tension tests on aluminum shafts which clearly show the superiority of another theory; and

II. A comparison of cement mortar columns with and without greased ends which, at best, points toward the Maximum Strain Theory as a plausible possibility.

The paper concludes with the suggestion that compression tests of concrete should be made on greased specimens, or a factor should be introduced to give equivalent values. Since all concrete column tests invariably include the making and testing of control specimens to measure concrete strength, it would appear that all necessary factors are included automatically.

Corrections for *Transactions:* In February, 1943, *Proceedings*, ordinate caption for Fig. 2, change "$\frac{e_n}{e}$" to "$\frac{\delta_n}{\delta}$"; in Eq. 7, change "$1 - \mu$" to "$1 + \mu$"; and, in Eq. 14*b*, change "$\frac{1}{2} + \sin\alpha\cos\alpha$" to "$\frac{1}{2} - \sin\alpha\cos\alpha$."

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[2] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1092.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DISTRIBUTION GRAPHS OF SUSPENDED-MATTER CONCENTRATION

Discussion

BY MESSRS. G. C. DOBSON, THOMAS R. CAMP, AND FRANK S. BAILEY

G. C. DOBSON,[4] M. AM. SOC. C. E.[4a]—A reasonably accurate estimate of the total suspended load of fine material transported by a natural stream involves a discouragingly expensive program of observations and sampling over a long period of time. Any suggestions for reducing the length or cost of these sampling programs without material loss of accuracy are very welcome. Professor Johnson's paper is a valuable contribution in this respect. Although his method depends on the usual field program until sufficient data are available for the development of a set of distribution graphs, the requirement thereafter of only one observed suspended-matter concentration for each rise or flood would reduce the cost materially and would eliminate some troublesome field problems. The assumptions involved seem in general to be as reasonable as those supporting the use of the "unit graph" which supplied the idea underlying Professor Johnson's method.

In the literature on transportation of sediment there are many references to the fact that no definite relation can be found between discharge and the load of suspended sediment. Unfortunately, little information is given as to why this relationship cannot be found. The author answers this question definitely and satisfactorily by citing the different factors affecting the availability and entrainment of the fine and coarse material moving as suspended load. It follows from this that a stream rarely, if ever, transports suspended matter to its capacity. In fact, it has not been demonstrated that a stream has a definite transporting capacity.

In the discussion of the two component parts of suspended load and the factors governing their availability, entrainment, and transportation, some implications, not developed by the author, seem too important to pass without discussion. If the wash load, which can constitute as much as 90% of the total suspended load, is carried in relatively permanent suspension and is

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NOTE.—This paper by Joe W. Johnson, Assoc. M. Am. Soc. C. E., was published in October, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1943, by H. A. Einstein, Assoc. M. Am. Soc. C. E.

[4] Senior Soil Conservationist, SCS, Greenville, S. C.

[4a] Received by the Secretary February 10, 1943.

composed of very fine material (finer than any found in appreciable quantities in the stream bed), practically all of this wash load must move continuously downstream during a flood, in suspension, at a velocity practically equal to that of the stream until it is deposited on parts of the flood plain where the velocity is greatly reduced, in a reservoir, in a lake, or in some other artificial or natural silt trap.

From the author's statements it seems logical to conclude that a very large part of the sediment entering a reservoir during a flood is composed of particles that had traveled from the points where they entered the stream channel to the reservoir in one continuous journey. The remainder of the sediment—much the smaller part—is made up of bed material in suspension and bed load, each governed by its own law of transportation, the particles traveling intermittently and requiring a long time to make the journey. This time factor should be of considerable importance to those interested in conserving bottom agricultural land, the capacities of reservoirs, or other downstream values because it means that any measures that would prevent the fine sediment, wash load, from entering the river system would be very efficient and immediate in their effect. There exists a rather prevalent belief that, because of the enormous amount of sediment now in river channels, such preventive measures would become effective only after a long period of time.

Thomas R. Camp,[5] M. Am. Soc. C. E.[5a]—The labor of measuring the silt load carried by a stream may be greatly reduced without undue sacrifice in precision, by the method presented in this paper. In regions subject to soil erosion, measurements of silt transport are necessary in order to gage the effectiveness of works constructed to reduce soil erosion. If the program is to be effective, a large number of stations must be established on streams and tributaries, and continuous records should be kept of the rate of flow of "farm land" toward the ocean. As a conservation measure, this program is just as important as stream gaging. The author is to be commended, therefore, for furnishing a means by which more stations may be operated for the same cost.

It has been found convenient, in Professor Johnson's experience, to separate "suspended load" from "bed load" and virtually to ignore the latter. Since it is the fine silt from farm land which is of interest in the conservation program, that is what should be measured. A word of caution is in order, however, concerning the temptation to draw conclusions regarding the mechanics of sediment transport from observations made in the field. The material that appears to be permanently in suspension is there because of its size and not because of its origin; and the material that appears to be transported only by bed-load movement remains at the bottom because of its size and not because of its origin.

The mechanics of the transport process is the same for all sizes of materials, except for the changes in drag coefficient of the suspended particles with size. The material that moves only along the bottom does so because the transport capacity of the eddies in the stream is inadequate to move it upward. The finer material which appears to be permanently in suspension is not permanently in suspension; it is continuously settling out (as it must if it is heavier than water) and is continuously being carried upward by turbulence.

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[5] Associate Prof., San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[5a] Received by the Secretary March 15, 1943.

The entire process of transportation, both on the bed and in suspension, depends for its continuance upon scour from the bed. There is no such thing as transportation in suspension, as such, for particles whose density differs from that of water. William E. Dobbins (12),[5b] Jun. Am. Soc. C. E., has demonstrated, both by theory and experiment, that material which appears to be permanently in suspension will settle out readily, under the same conditions of turbulence in the stream, if scour from the bottom is prevented. The equilibrium condition in which the concentration of suspended matter at any depth remains substantially constant with time can be maintained only if the rate of settling out across any horizontal plane—including the bottom—is exactly balanced by the rate of upward transport of sediment.

The mechanics of turbulent transport of sediment in suspension has been studied widely and is understood fairly well. A. Shields (13) has presented a satisfactory rational development for the critical tractive force at which bed-load movement starts. No satisfactory attack, however, has yet been made on the problem of the relations between the rate of scour from the bed and the hydraulic characteristics of the stream. Many measurements have been made, in the laboratory, of the transport capacity of streams, but, since nearly all the studies have been confined to bed-load movement, little information is available on the total transport capacity of a stream. For a better understanding of the hydraulic conditions governing the rate of scour, a combined theoretical and laboratory study is needed.

FRANK S. BAILEY,[6] ASSOC. M. AM. SOC. C. E.[6a]—The use of a distribution graph of suspended-matter concentration is described in this paper together with the observed hydrograph and a single observed suspended-matter concentration for calculating the total suspended load of fine material transported during a particular storm. These phenomena deserve the attention of all persons interested in the collection of such data.

Having noticed the resemblance of distribution graphs of different storms for a particular stream, Professor Johnson shows quite convincingly that sufficiently accurate results may be obtained at much less labor and expense than formerly have been considered necessary.

In these days, especially when so many men are being taken into the armed services, a method of obtaining needed information, which eliminates much unnecessary effort, should be utilized whenever possible. Of course, the various changing conditions on watersheds which the author mentions always should be considered in applying his method.

Extraordinary storms may occur in which the single observation may be found inadequate, but for average cases the writer believes Professor Johnson has developed a valuable idea, which should be tested carefully in various localities.

[5b] Numerals in parentheses, thus: (12), refer to corresponding items in the Bibliography, in the Appendix of the paper, and at the end of discussion in this issue.

[6] With Stone & Webster Eng. Corp., Boston, Mass.

[6a] Received by the Secretary March 16, 1943.

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(12) **"Effect of Turbulence on Sedimentation,"** by William E. Dobbins, *Proceedings*, Am. Soc. C. E., February, 1943, p. 235.

(13) **"An Analysis of Sediment Transportation in the Light of Fluid Turbulence,"** by Hunter Rouse, Sedimentation Div., SCS, U. S. D. A., July, 1939, p. 5.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RIVETED AND PIN-CONNECTED JOINTS OF STEEL AND ALUMINUM ALLOYS

Discussion

BY JONATHAN JONES, M. AM. SOC. C. E.

JONATHAN JONES,[15] M. AM. SOC. C. E.[15a]—The only question the writer would ask with respect to the program as a whole is: If the objective was (as stated in the "Introduction") to investigate longitudinal distribution of rivet loads, would it not have been more convincing to arrange the twelve rivets on each side of a joint in two lines of six each or three lines of four each, instead of in four lines of three each, the plates to be correspondingly narrower and thicker? All further comment herein is directed solely to the tests of carbon steel plates.

The reported shearing strength of rivets (41,100 lb per sq in. in Table 8 and 40,600 in Table 9(*a*)) appears to be lower than may be expected in hot-riveted work. It should be noted that these rivets were not hot-driven, and their strength therefore was not enhanced by the usual driving "quench."[16] Neither was their strength enhanced by cold work, as in standard cold driving, since they had a press fit in the holes and the work of cold upsetting must have been substantially confined to the heads.

One of the most important findings to record from a test of a riveted joint is the ratio between the shear strength of the driven rivet and the tensile strength of the original rivet bar.[17,18] In the paper the latter value is not given, so it cannot be determined whether, as the writer has just intimated, this important ratio was low. If any of the original materials are still at hand it would be very interesting if the ratio in question could be recorded for (*a*) the rivets as driven and recorded in the paper, (*b*) rivets driven hot, and (*c*) rivets

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NOTE.—This paper by Leon S. Moisseiff, M. Am. Soc. C. E., and E. C. Hartmann, and R. L. Moore, Assoc. Members, Am. Soc. C. E., was published in January, 1943, *Proceedings*.

[15] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

[15a] Received by the Secretary March 4, 1943.

[16] "Static Tests of Riveted Joints," by Jonathan Jones, *Civil Engineering*, May, 1940 (see heading, "Strength of Rivets in Shear"), p. 286.

[17] *Ibid.*, p. 287.

[18] "Physical Properties of Driven and Undriven Rivets of High-Strength Structural Steels," Progress Report of the Committee of the Structural Division on Structural Alloys, *Proceedings*, Am. Soc. C. E., May, 1942, p. 769, Table 13, Col. 26.

driven by the usual technique of cold driving in which the shank is $\frac{1}{16}$-in. slack in the hole before upsetting.

In sum, it is felt that the results reported in this paper should not influence the rivet shear allowance in specifications based on standard structural practices.

In the test designed to fail in bearing, the actual failure (see Fig. 15) was not a bearing failure, which implies collapse on the contact surface between pin and plate, but a bulging out behind the pin through a splitting or shearing action, accompanied by a transverse tension tending to rupture the free end of the plate. Evidently the greater the length of plate behind the pin, the more resistance there would be to this effect, and the higher the true bearing pressure could be raised. Had the length behind the pin in this test been about 4 in. instead of 3 in., it is probable, judging by other tests, that the load could have been increased and that there then would have been about an equal chance of failure of the type that did occur, and of failure in tension across the pinhole. A true bearing failure, meaning a crumpling of plate material at point of contact with the pin, is probably not attainable under ordinary relationships between thicknesses of plate and diameter of pin or rivet, although some thickening of the plate will be observed.

In the tests reported[17] by the writer in 1940 the rivet bearing reached 165,000 lb per sq in. before joint yield was observed, and 204,000 lb per sq in. before failure occurred. The authors' test evidently did not exhaust the bearing capacity of the steel plate, despite the fact that the unit bearing stress at failure was 3.3 times the American Institute of Steel Construction (A.I.S.C.) allowance for enclosed rivet bearing and 4.1 times the A.I.S.C. allowance for non-enclosed rivet bearing or for bearing on pins.

In all pin-connected or riveted construction, due attention should be paid to the length of plate behind the pin or rivet. Tests have indicated that if a pin-connected flat plate of width b has a net width across the pin of 1.30 b and a net length behind the pin of 0.85 b, failure will occur in the bar and not in the head.

Corrections for *Transactions:* In January, 1943, *Proceedings*, page 4, Figs. 1(*a*) and 1(*b*), in the pin openings, the word "shackle" should be "shackle pin"; in Table 1, change footnote [a] to read "27S–T, 17S–T, and 53S–T are aluminum alloy and temper designations," under "Description" Item 1 should read "Rivet or Pin Material," and in footnote [d] "53S–T" should be "53S–W"; on page 10, insert footnote index[7] after "material" in line 38, and in footnote [7] delete "ASTM–E8–33" and substitute "'Structural Aluminum Handbook,' Aluminum Co. of America, Pittsburgh, Pa., 1940"; on page 11, change the first two sentences to read "In the foregoing list, all specimens were proportional to that shown in Fig. 3,[8]" and run in with the following paragraph; in Table 2, Col. 1, delete "ASTM"; on page 12, line 20, change "six ½-in." to "6.5-in."; on page 12, line 34, change "each" to "one"; in Table 3, change footnote [b] to read "The rivets and pins in this group were 53S–W"; on page 17, caption to Fig. 6, delete the word "steel"; on page 18, line 1, change "Table 4" to "Table 5"; and, in Table 9, heading for Col. 10, change "shear" to "ultimate."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RELATION OF UNDISTURBED SAMPLING TO LABORATORY TESTING

Discussion

BY MESSRS. RAYMOND F. DAWSON, AND HAMILTON GRAY

RAYMOND F. DAWSON,[6] M. AM. SOC. C. E.[6a]—Because the expansive clays of the Southwest present unusual problems as foundation materials, a comparison of these soils with those included in Professor Rutledge's paper is of interest. The writer has never found an undisturbed sample with all the voids filled with water. The initial degree of saturation usually ranges between 94% and 99% and has been found as low as 89%. Since many of these samples were taken at a considerable depth below the surface, where ample water should have been available to fill the voids, this indicates a swelling of the specimen. Perhaps the initial degree of saturation is a measure of disturbance in these clays.

These samples continue to expand at a very slow rate after they have been removed from the ground. Specimens covered with paraffin and stored in the moist room will eventually expand enough to break the covering. With some samples this will occur in only a few weeks whereas others require many months to produce failure. This condition has caused local laboratories to apply two special procedures to the consolidation test: Samples are tested as quickly as possible after being removed from the ground; and the initial void ratio is held constant until sufficient load has been applied to offset the swelling pressure. The latter step is accomplished by applying small increments of load to the specimen after it has been placed in the consolidometer, the amount of load applied being sufficient to prevent movement of the dial gage. When no further load is required to keep the dial constant, the sample is loaded in the usual manner, starting with the next increment of load generally used in the consolidation test. On the pressure, void-ratio, curve this part will be a straight line since the void ratio is constant and is usually indicated by a broken line.

NOTE.—This paper by P. C. Rutledge, Assoc. M. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Messrs. Benjamin K. Hough, Jr., and F. M. Van Auken; and March, 1943, by Jacob Feld, M. Am. Soc. C. E.

[6] Associate Prof., Civ. Eng. Testing Engr., and Asst. Director, Bureau of Eng. Research, Univ. of Texas, Austin, Tex.

[6a] Received by the Secretary March 4, 1943.

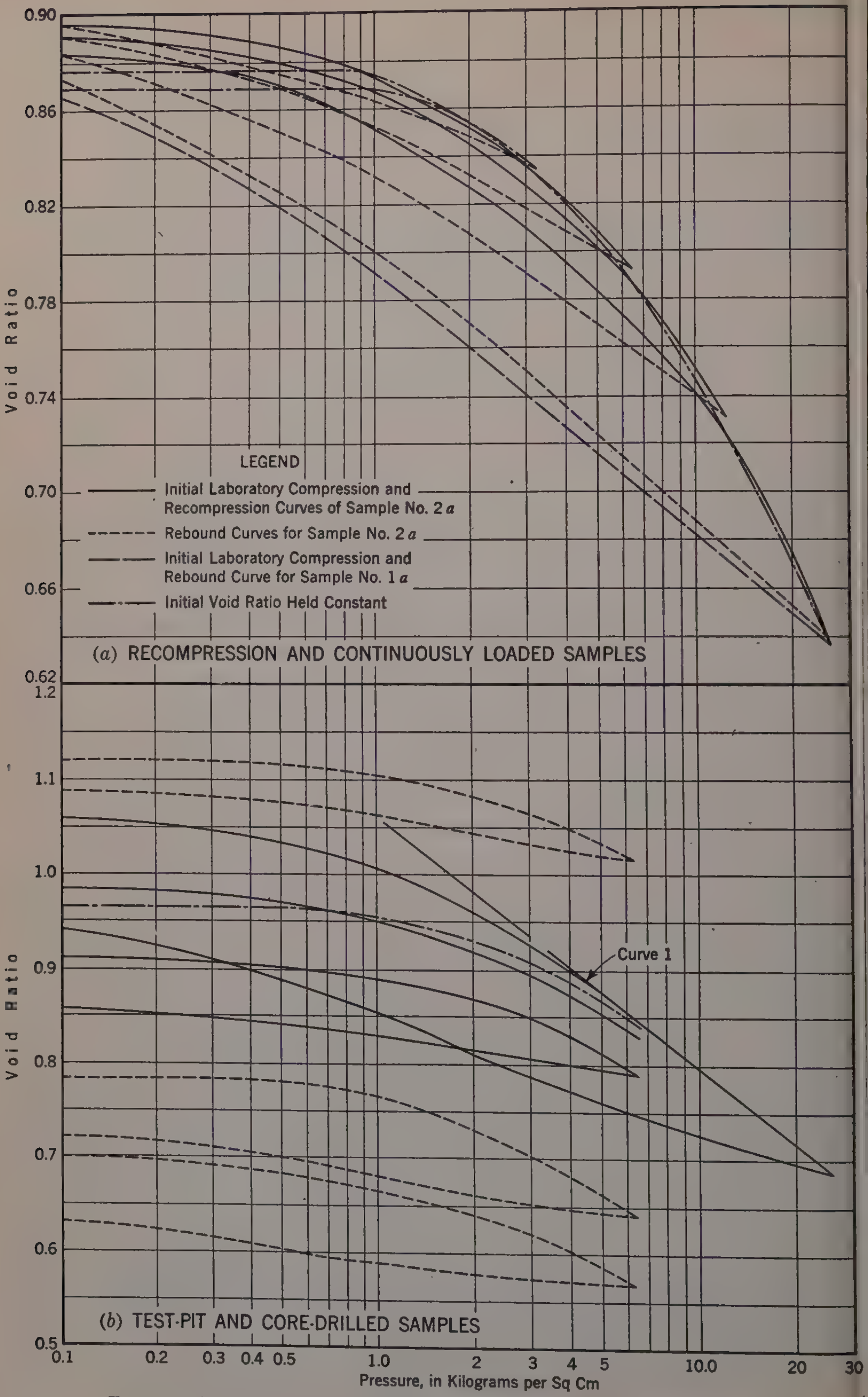

FIG. 12.—EFFECT OF SWELLING ON THE CONSOLIDATION CHARACTERISTICS OF EXPANSIVE CLAYS IN THE SOUTHWEST

In order to study the effect of swelling on the consolidation characteristics of these soils the test shown in Fig. 12(*a*) was made. A sample was prepared in the usual manner and the initial void ratio was held constant, as indicated by the dash-dot line. The soil was then loaded in increments to a total pressure of 3.2 kg per sq cm and the load removed to produce the rebound curve. With a load of 0.1 kg per sq cm (200 lb per sq ft) the specimen had expanded considerably above its original void ratio and the recompression curve started. In each cycle of recompression the maximum load was twice that used in the previous cycle. The recompression curves indicate that each cycle of loading changes the slope of the lower portion of the curve in a downward direction. The results of a consolidation test, made in the usual manner on a sample of similar soil, are shown in Fig. 12(*a*) by the dashed line. This indicates that the change in character of the curves due to recompression is not as important as the recompression curves alone would lead one to believe. The soil used in the recompression tests (sample 2*a*) had an initial void ratio of 0.869 and initial degree of saturation of 98.5%, whereas the continuously loaded specimen (sample 1*a*) had an initial void ratio of 0.877 and an initial degree of saturation of 97.5%. Continuous loading tests on other samples from the same site gave pressure, void-ratio, curves with shapes similar to sample 1*a* but with higher or lower initial void ratios.

Although it is agreed that cubical samples, taken carefully from a test pit, give more nearly undisturbed test specimens than those obtained by any other method, it may not be possible economically to use this procedure, especially in the case of a deep stratum. In 1942, the writer supervised the sampling of clay soils on two projects where it was impracticable to use test pits and where no special sampling spoon was available. On both jobs samples were taken by using what the drillers call a "poor-boy" sampler. This sampler consisted of a piece of seamless steel tubing, $\frac{1}{8}$ in. thick, with a 5-in. outside diameter. The lower edge was sharpened, but the 12-in. long tube had the same internal and external diameters for the entire length. Holes, 6 in. in diameter, were advanced by usual drilling methods until the desired depth was reached at which point the sampling tube was inserted for the drilling bit. The sampling tube was forced into the soil for a depth of 8 to 10 in. by means of a hydraulic jack and the bit rotated one turn to shear off the soil at the bottom of the bit. When it was necessary to advance the hole by wet drilling, the top 2 to 3 in. of the core was removed and discarded. As soon as the cores were removed from the tube, they were coated with several layers of paraffin and cheesecloth and tested immediately after being transported to the laboratory. These sampling tubes had an area ratio (9)[6b] of less than 10% as compared with about 40% for the usual sampling spoon. Therefore, they probably caused less disturbance than some of the more elaborate sampling devices. On one project a seamless steel tube only $\frac{1}{32}$ in. thick was used part of the time. This gave an area ratio of about 5% and proved satisfactory until a pebble which forced it out of shape was encountered. Visual examination showed very little disturbance of cores taken by this method.

[6b] Numerals in parentheses, thus: (9), refer to corresponding items in the Bibliography (see Appendix) of the paper, and at the end of discussion in this issue.

On one project the investigators were able to compare the results obtained on cores taken as outlined, with the consolidation characteristics of the same soil as determined from cubical samples cut carefully from test pits; Fig. 12(*b*) gives this comparison. The solid-line curves were obtained for specimens taken from the test pits and the broken-line curves were for the core-drilled samples. There is little difference in the shape of the curves, indicating either (*a*) that there was practically no disturbance in taking the samples with the core drill; or (*b*) that the entire soil deposit is in a disturbed condition.

A further examination of all the curves shown in Fig. 12 might lead also to the conclusion that the soils were disturbed because they do not possess the general shape indicated by Professor Rutledge and most of them do not have a straight-line section. This is a result of the type of soil rather than disturbance at the time of sampling. When high consolidation loads are used on these soils, the lower portion does become a straight line as shown on curve (1), Fig. 12(*b*). Such a curve indicates a preconsolidation load of about 2 tons per sq ft, which is about twice the weight of the overburden; and, since the clays are of recent geological origin, one would not expect that any great amount of overburden has been removed. These clays have been badly fissured and break with slickensided surfaces, many of which contain thin layers of clay of a different color. It is assumed, therefore, that the clay dried and cracked after it was deposited and that the next flood filled the cracks with the lighter-colored clay; then, when the original clay expanded by taking up moisture, internal stresses developed because the cracks were filled, therefore causing the high preconsolidation load.

Engineers and investigators should underline Professor Rutledge's statement (paragraph following Eq. 4):

> "Cautious and intelligent use of theoretical conceptions and methods is the hope of the future in foundation engineering. The restrictive assumptions of rigorous theoretical methods mean that they can never be applied safely to soils without full-scale field observations to determine their limitations."

Certainly many more full-scale field observations accompanied by complete laboratory analyses must be conducted before final conclusions may be reached.

HAMILTON GRAY,[7] JUN. AM. SOC. C. E.[7a]—The principal steps involved in a foundation analysis have been aptly likened by the author to "links in a chain" of procedures which are intended to reach a definite point that may be considered the solution of the problem at hand. Conditions which produce inaccuracies in any link—figuratively changing the length of the link—tend to affect the length of the chain and hence the distance by which it fails to reach the desired terminal point. Such inaccuracies, of course, may be compensating, the effect of a "short" link being offset by that of a "long" one; but such an occurrence would be purely fortuitous. A close agreement between predicted and observed behavior associated with any physical phenomenon does not necessarily assure that each of the steps involved in making the prediction was flawless. Similar methods may not give as "satisfactory" results when applied another time. Hence, if the methods of soil mechanics are to achieve their full usefulness, the magnitude of the inaccuracies in the individual links must be known quantitatively so that the error in the length of the chain may be determined with reasonable certainty. Actually this chain consists of three major links, which, together with the chief source of error in each, may be listed as follows:

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[7] Asst. Prof., Civ. Eng., New York Univ., New York, N. Y.

[7a] Received by the Secretary March 9, 1943.

(1) Sampling (distortion of the soil and hence alteration of its physical properties during the sampling process);
(2) Testing (inappropriate testing methods for determining the pertinent physical properties); and
(3) Prediction (unsuitable analytic methods for predicting the behavior of the underground on the basis of values determined under Item 2).

If one were in a position to evaluate, confidently, the magnitudes of the errors inherent in each of these three steps, the science of foundations would be practically complete. Professor Rutledge has made an encouraging start toward determining the effect of imperfect sampling on certain of the physical characteristics which are determined by experimental means. It is clear, however, that the numerical and graphical comparisons are made between samples, and not between samples and the soil in place. Consequently, it remains difficult to know quantitatively how truly a sample represents its parent stratum.

Only by comparing properly conducted field observations with the solutions secured by means of steps 1, 2, and 3 in a large number of cases can one ever hope to determine the sources of discrepancy between predicted and actual observed behavior. In the present state of knowledge, an agreement between prediction and field behavior does not assure perfection of all links in the chain; nor may a discrepancy between prediction and behavior definitely be attributed to any one of them. One simply cannot state, in the light of present knowledge, what quantitative influence is exerted by inaccuracies in any or all of the three steps enumerated. Hence, only rough estimates can be made of foundation behavior.

The paper does suggest means of forming qualitative opinions as to the possible value of a sample in representing the stratum from which it was secured, and shows, further, that a given degree of soil disturbance influences certain physical properties more than others. This signifies, for example, that great caution should be employed in utilizing the modulus of elasticity as obtained from unconfined compression tests. On the other hand, the compressive strength secured in the same test is more nearly representative of the behavior in nature. Although it is easy to establish a numerical measure giving the degree of disturbance intermediate between complete remolding (100%) and the best available undisturbed sample (0%), comparison with the behavior of the parent stratum remains a matter of guess or judgment.

In connection with the consolidation test, the author leaves the impression that a pressure, void-ratio curve for a loose clay, which shows upward concavity (see Fig. 5) instead of being precisely linear (see Fig. 2), when plotted on semilogarithmic paper represents a minimum amount of disturbance. The writer has observed such upward concavity in loose clay samples of 2.5-in. diameter, which were taken in a sampling spoon (3-in. inside diameter) of a type no longer considered suitable for securing first-class undisturbed samples, and believes that there was no minimum of disturbance in these cases. Further, since a sample taken by spoon is generally subject to the greatest amount of distortion at its periphery, one is led to consider the extreme case of a "composite" sample consisting of a wholly undisturbed core of diameter, say 3 in., surrounded by a completely remolded soil annulus of the same material to give a total sample diameter of 4 in. When subjected to a consolidation test, the deformation of both core and soft annulus will be identical, but the annulus will be subject to a far lower stress because of its different pressure, void-ratio diagram (see Figs. 3, 4, and 6). An analysis of the loading and deformation of such a sample leads to the conclusion that the resultant pressure, void-ratio diagram would be slightly concave upward at the lower end just as are the curves in Fig. 5. Until further information is available, therefore, it would seem wise to accept, with definite reservation, the author's statement concerning the evidence of a minimum of disturbance. The foregoing inference should be easy to check experimentally for the behavior of composite samples of various clays.

In discussing the triaxial compression test the author touches upon the somewhat controversial subject of the proper type of test to be made in simulating certain field conditions. Similar differences of opinion exist in connection with quick and slow shear tests. The type of test to be used should be one which either simulates field behavior, or gives results that may be used somehow in analyzing conditions of soil in place. If two or more kinds of tests can furnish usable results in a given set of circumstances, that one which is performed more readily probably will be used the more widely. On the other hand, the method of testing always must be adapted to the type of analysis employed in predicting field behavior. For example, the author cites Eq. 4 for computing settlements. The use of this formula entails the previous determination of the soil properties, defined by K and E, which in turn require two types of tests. There are various other ways of estimating settlements, each with its proponents, and each necessitating knowledge of certain physical properties, determination of which may require more, or less, experimental work than do the constants of Eq. 4, depending on the details of the method. In all probability, the method that is best never can be decided by theory alone; but, in time, the compilation of observed behavior may give a clue as to the most reliable method.

A compilation of observational data of similar nature from several foundations which differ somewhat in such details as—(*a*) relative positions and thicknesses of soil strata; (*b*) size of loaded areas; (*c*) increase in net load; and (*d*) sampling technique—might permit a segregation of the inaccuracies inherent in the various steps involved in making a forecast of behavior. The data from a single foundation may or may not suffice to justify assigning the source of any discrepancy between prediction and observation to one of the steps employed in making the prediction. To cite a specific example, assume that a building imposing a uniformly distributed net increase in pressure of 1 ton per sq ft at the level of its foundation rests on a bed of dense sand. The sand extends to a depth of 15 ft below the foundation and is underlain by compressible clay overlying rock. If the building covers a very large area and the clay stratum is thin, any variation between observed and predicted subsidences may be attributed to disturbance during sampling or to improper testing technique with the former constituting the more probable source. On the other hand, if the area covered by the building is relatively small and the clay stratum thick, the variation of pressure increase with depth is introduced. This brings up the question of the applicability of the "Problem of Boussinesq," and, in addition, the more difficult matter of the effect of the sand layer in distributing loads to the more compressible clay. This latter problem in all probability cannot be treated satisfactorily by analysis alone, and hence it appears evident that a widespread collection of observational data, followed by an intensive study of its significance, would be required before an understanding of such conditions is reasonably complete. However, as suggested herein, a relatively small amount of data probably would suffice, under favorable conditions, to determine the influence of sample disturbance on the laboratory test results alone. That modification of the test results requisite to predict the actual behavior of the foundation then could be adopted readily. However, without the correlation between laboratory results and observed field behavior, the magnitude of the desired modification cannot be known with any certainty.

Bibliography.—

(9) **"The Present Status of the Art of Obtaining Undisturbed Samples of Soils,"** by M. J. Hvorslev, preliminary draft of report of Committee on Sampling and Testing, Am. Soc. C. E., March, 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

SEISMIC SUBSURFACE EXPLORATION ON THE ST. LAWRENCE RIVER PROJECT

Discussion

By R. Woodward Moore, Esq.

R. Woodward Moore,[5] Esq.[5a]—The application of refraction seismic tests to shallow explorations in water-covered areas constitutes a distinct forward step in the use of geophysics on civil engineering projects. This method of test already has proved to be a useful tool in connection with subsurface explorations for dam sites, highway grading projects, rock quarry sites, and tunnel locations. Data concerning foundation conditions at bridge abutment locations have been obtained with seismic tests, and, by means of subaqueous tests such as are described by the authors, it should be possible to investigate pier locations as well. The value of the application of the seismograph to bridge site explorations should not be underestimated, particularly in preliminary surveys for highways through undeveloped areas not adequately served by existing roads or trails. At times, a rapid survey of subsurface conditions using the seismograph may eliminate the need for further check tests using the more costly and time-consuming core drill or test pit. Although the exploration costs for a bridge structure may be relatively small, the ratio of cost of exploration to total cost of the structure may be relatively large in comparison with that for a large dam or reservoir.

In connection with the tests on land in the Point Rockway Canal area where the drill located a solid rock formation with a shallow overburden and where the seismic data were inconclusive, it would have been informative to have made additional seismic tests with the detectors and shot lines at right angles to the lines that were used. In one or two instances in the writer's experience appreciable differences in wave velocity (3,000 to 4,000 ft per sec) have been obtained when shooting two lines of tests oriented 90° with respect to each other. One location where such differences in wave velocity were found was near the bank of a small river. It is possible that early stream

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Note.—This paper by E. R. Shepard, Esq., and Reuben M. Haines, Assoc. M. Am. Soc. C. E., was published in December, 1942, *Proceedings*.

[5] Associate Civ. Engr., FWA, U. S. Public Roads Administration, Washington, D. C.

[5a] Received by the Secretary March 1, 1943.

action exposed the ledge rock to the elements, which resulted in weathering characterized by vertical jointing or fracture zones. This is entirely suppositional since no rock was exposed at the surface at this location. Concealed by a mantle of soil deposited during subsequent periods of erosion, these fracture zones would tend to lower the wave velocity in the rock for seismic tests made along lines transverse to the weathered zones. Tests along lines parallel to the fracture zones would result in wave velocities characteristic of a fairly solid, continuous mass of rock.

With respect to seismic tests made on frozen ground, probably a carbon-button type detector such as was used in the tests on the St. Lawrence River Project, and which has been used by the writer, could be adjusted to such a degree of sensitivity that the first impulse or wave through the frozen ground would produce a relatively small break in the oscillograph trace. The slower but stronger impulse passing through the normal soil below the frozen zone then would be present on the oscillograph record as a "second event" which could be interpreted as the true time of wave travel from the shot to the detector. These "second events" have been used by the writer in the interpretation of seismic records which were obtained in areas where a low-velocity layer was underlain by a rather thin stratum having an intermediate wave velocity which rested upon solid rock. Fig. 13 shows a film record obtained with the carbon-button detector in which "second events" are shown rather clearly.

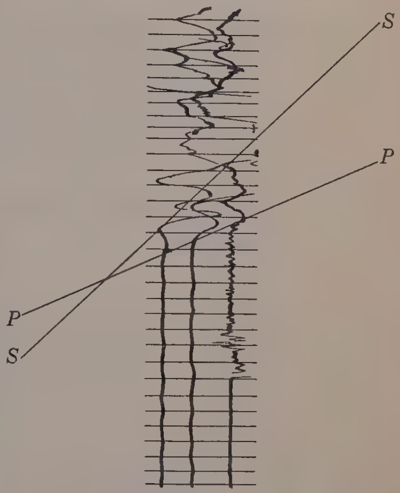

FIG. 13.—THREE-CHANNEL SEISMIC RECORD SHOWING PRIMARY (P–P) AND SECONDARY (S–S) WAVE ARRIVALS

Referring to the tests made "in quiet or slowly moving water," it would be interesting to know the actual velocity of the slowly moving water since there is some likelihood of water currents disturbing the detectors and causing troublesome vibrations which might mask the seismic wave coming from the shot. Research work may be required to determine the effect of various water velocities upon the submerged detector units, particularly where pipes or rods which extend to the surface of the stream are used. The effect of the flow conditions at the bottom of swift but deep water courses may require some remedial measures to assure a "quiet" detector should a satisfactory means of placing the units in swift water be developed.

In their analysis of the data obtained from the tests in swift water, the authors point out the probable sources of error in determining the elevation of the rock at the shot point in the stream bed. The average of the data from several shots at a particular point would tend to reduce the error in obtaining t_1 for the time of wave travel in the material overlying the rock. However, rock elevations obtained in the manner described should be useful in the design of dams and bridge piers.

In general, the accuracy indicated in a comparison of the rock elevations predicted from seismic data and those obtained from direct drilling, as given in Table 1, is in close agreement with the degree of accuracy obtained in similarly correlated tests made by the U. S. Public Roads Administration during the eight years since 1935. Usually the rock surface could be predicted within 5% to 10% of the true depth.

Corrections for *Transactions:* In December, 1942, *Proceedings*, page 1749, change the last two lines to read “$V_r = \frac{2 \times 24{,}600 \times 12{,}300}{24{,}600 + 12{,}300} = 16{,}400$; $\sin \alpha = \frac{V_1}{V_2}$, and $\cos \alpha = \sqrt{1 - \left(\frac{V_1}{V_2}\right)^2} = 0.97$; and, $\sin \beta_2 = \frac{V_2}{V_r}$, and $\cos \beta_2 = \sqrt{1 - \left(\frac{V_2}{V_r}\right)^2} = 0.925$. Then: * * *”; in Fig. 5 change “$V_r = 1{,}700$” to “$V_r = 17{,}000$-rock”; and, on page 1760, line 12, change “318” to “323.”

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ORGANIZING AND FINANCING SEWAGE TREATMENT PROJECTS

Discussion

BY HERBERT MOORE, ASSOC. M. AM. SOC. C. E.

HERBERT MOORE,[5] ASSOC. M. AM. SOC. C. E.[5a]—The organization of large sanitary districts is the subject of this paper, such districts being required because of the "magnitude and difficulty" of projects for sewage treatment. Often, however, a need for relatively simple sewerage improvements can be met by the organization of a small sanitary district.

In Wisconsin, enabling legislation makes this possible. Some forty sanitary districts have been organized under Wisconsin statutes 60.30 to 60.309, in unincorporated township areas. Districts have been formed for the most part to provide sewers only, in order to discharge sewage into the system of a neighboring incorporated community or metropolitan sanitary district. However, several small sanitary districts provide their own sewage treatment.

The first step in the organization of a sanitary district is a petition to the town board signed by at least 60% of the property owners of a given area. Following a public hearing, the town board establishes the boundary and appoints three commissioners at a nominal fee (generally about $25 each per year) to administer the district. The district may then construct sewerage improvements and finance the project by bond issue.

Thus, a sewerage and sewage disposal system serving a small suburban residential community in Metropolitan Milwaukee (Wis.) was constructed by the Westchester Sanitary District, Town of Brookfield, Wis., Waukesha County, and placed in operation in December, 1941. This project provides the main sewers, a sewage disposal tank, and sewer connections from house building lines. It is believed to be the first public sewerage and sewage disposal system to be constructed from homes through the disposal system complete. Furthermore no federal aid was received.

Located about one mile west of the limits of the area served by the Metropolitan Sewerage Commission of Milwaukee County, the Westchester Sanitary

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NOTE.—This paper by Samuel A. Greeley, M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by Messrs. Milton P. Adams, and Hal F. Smith.

[5] Cons. Engr., Milwaukee, Wis.

[5a] Received by the Secretary February 13, 1943.

District was formed by the homeowners and residents of a housing community to correct nuisance conditions resulting from the use of septic tanks. Some forty houses had been constructed and septic tanks and gravel leaching beds were installed in accordance with best approved practice. However, soon after the houses were occupied, septic sewage effluent from the leaching beds emerged upon the surface of the ground, flowing into ditches and across neighboring property, thereby causing considerable odor and nuisance. The organization of the sanitary district resulted from the demand by property owners for immediate corrective measures.

Plans and specifications were prepared and advertised. The contract was awarded to the lowest of six bidders and the system, comprising some 8,000 ft of sewer lines and a concrete two-story-type tank, was constructed. Sewage effluent from the tank flows into a gravelly subsoil on the property of the District. Six months' time elapsed between the formation of the Sanitary District and the completion of construction of the project.

Work was financed by general obligation bonds in the amount of $4,200 and revenue bonds in the amount of $12,500, at interest rates of 2.4% and 4.5%, respectively. The general obligation bond issue was limited by statute to 5% of the assessed valuation at the time of the last previous assessment.

Correction for *Transactions:* In December, 1942, *Proceedings*, page 1729, change the caption to read:

FIG. 1.—METROPOLITAN ORGANIZATIONS ARE THOSE COMPRISING A LARGE CITY, THE ADJOINING MUNICIPALITY, AND PERHAPS SOME AREA OF UNINCORPORATED LAND. THE BUFFALO (N.Y.) SEWER AUTHORITY IS A METROPOLITAN ORGANIZATION OF THIS TYPE. VIEW SHOWS CONSTRUCTION OF HARBOR CROSSING WHICH CONNECTS TO THE TREATMENT PLANT

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EARTH PRESSURE AND SHEARING RESISTANCE OF PLASTIC CLAY

A SYMPOSIUM

Discussion

BY MESSRS. A. E. CUMMINGS, AND D. M. BURMISTER

A. E. CUMMINGS,[28] M. AM. SOC. C. E.[28a]—The papers included in this Symposium contain some of the most worth-while information that has ever been published about the behavior of soft plastic clay. In addition, the correlations between laboratory tests and field measurements offer an excellent opportunity for comparing two fundamentally different methods of laboratory test procedure.

On the Chicago subway work, the shearing resistance of the clay was determined by means of unconfined compression tests. Soil samples for this test usually are obtained in thin-walled seamless steel tubes. In the unconfined compression test, a small cylinder of clay is placed in the testing machine and tested to failure just as a concrete cylinder is tested for its ultimate compressive strength. In some cases, the Chicago test specimens failed on a shear plane passing diagonally across the specimen. In most cases, the specimens did not fail completely but simply bulged like a barrel. Whenever bulging of this kind occurred, loading was continued until the height of the specimen was reduced by 20% of its original height. The shearing resistance was defined as one half the load at which the specimen failed completely or as one half the load that produced 20% strain in those specimens that bulged without failing.

On the Detroit tunnel work, the shearing resistance of the soil was determined by means of the ring-shear test. Soil samples for this type of test are obtained in the field in sectional brass tubes which are designed to fit the laboratory testing apparatus. The tube is about 7 in. long and is cut crosswise into three sections. The two end sections are each about 3 in. long and the center section is about 1 in. long. The tube containing a soil sample is placed in a horizontal position in the testing machine and the two end sections are clamped

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NOTE.—This Symposium was published in June, 1942, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1942, by Messrs. Ralph H. Burke, L. G. Lenhardt, George E. Shafer, and M. E. Chamberlain; and December, 1942, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[28] Research Engr., Raymond Concrete Pile Co., New York, N. Y.

[28a] Received by the Secretary March 3, 1943.

in place. Load is applied to the center section so that the soil sample is tested like a rivet in double shear. This type of test has shown that deformation under a given load practically stops in a few minutes when the load is very small. As the load is increased, deformation finally becomes progressive and the soil continues to yield without any increase in load. The test results are plotted in such a manner that the curves indicate the shearing resistance at which progressive yielding of the soil begins.

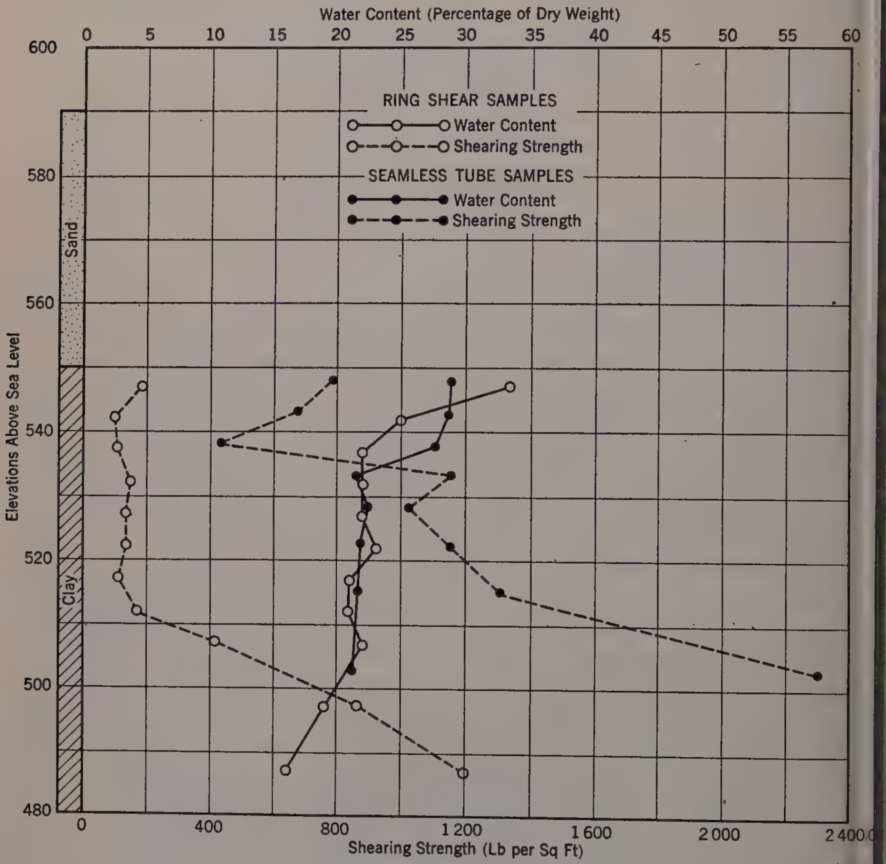

Fig. 48

It is seen readily that these two types of tests are fundamentally different sc that it would be a matter of considerable interest to determine what relationshi might exist between the shearing resistances determined by the two methods. It has been the writer's privilege to work with all three of the Symposium authors for many years, but he is familiar with only one case in which both type of test were made on soil samples secured from the same site. This project wa that near Indiana Harbor, Ind., on the south coast of Lake Michigan. On thi job a total of 120 samples were secured for ring-shear tests and 17 samples fo unconfined compression tests. The ring-shear samples were taken in the standard type of apparatus used by Professor Housel and the tests were performed in his laboratory. The samples for the unconfined compression tests were taken in seamless tubes by the same methods that were used on the Chicago subway and these samples were tested by Mr. Peck in his laboratory.

The soil conditions over the entire site are quite uniform. A bed of sand about 40 ft thick overlies a bed of plastic clay 60 to 70 ft thick. The upper part of the clay bed is quite soft but the clay becomes increasingly hard as the depth increases. Fig. 48 is a composite diagram showing the results of tests on samples taken from two borings which were about 75 ft apart. As far as natural water content is concerned, the two sets of samples differed very little. On the other hand, the shearing strengths determined by the two different test methods are vastly different. The unconfined compression tests on the seamless tube samples indicate a shearing strength that is very much greater than that determined in the ring-shear tests.

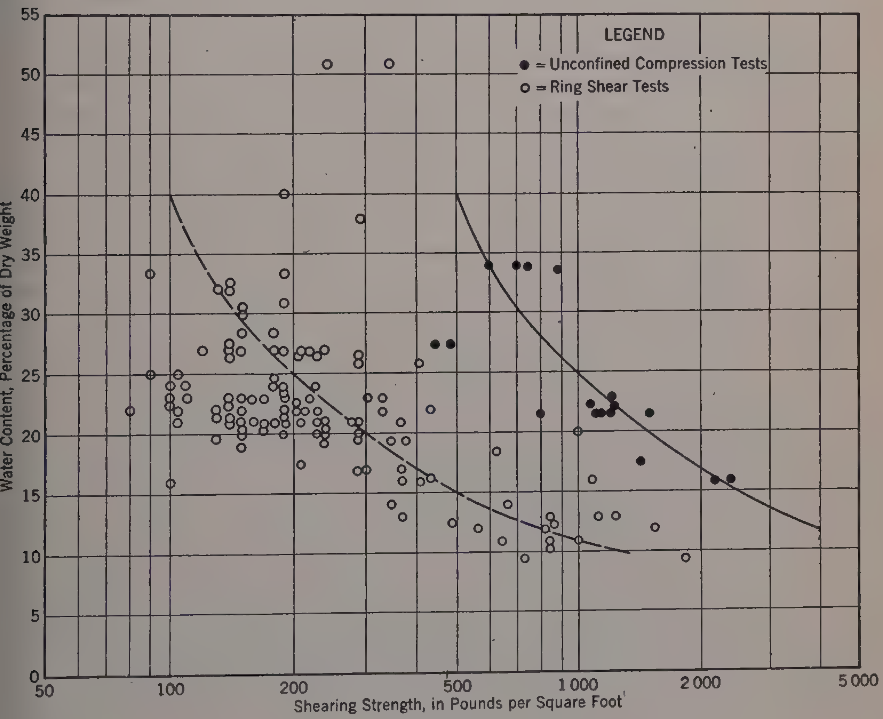

FIG. 49

In connection with the Chicago subway work, a rather elaborate statistical analysis was made to determine the relationship between the natural water content and the unconfined compressive strength.[29] Although the analysis indicated large standard deviations in the statistical relationship between these two variables, it did show that the results tended to cluster about an average value. Fig. 49 is a semi-logarithmic plot of the Indiana Harbor test results showing the relationship between natural water content and shearing strength. For a given water content, the shearing strength can vary over a wide range. However, the dotted curve is an approximate average of the results obtained from the ring-shear tests. For any given water content, the solid curve represents shearing strengths that are five times as great as those of the dotted curve. It is seen readily that the solid curve is a fair average curve for the results obtained from the unconfined compression tests. In other words, these two types of laboratory tests were made on soil samples taken from the same site and the average shearing resistance determined by the unconfined compression test was about five times as great as that determined by the ring-shear test.

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[29] *Proceedings*, Purdue Conference on Soil Mechanics and Its Applications, 1940, p. 149.

This wide difference in the test results can be explained as follows: The unconfined compression test determines a quantity that should be called the "ultimate shearing resistance" of the clay. The ring-shear test determines a quantity that should be called the "elastic limit" or the "yield point" of the clay. This fundamental difference raises the question of the relative merits of the two types of tests as a means of investigating the problems to which the test results are to be applied. In tunnel work and in large open cuts such as are described in the Symposium, there is always some flow in the clay bed. In other words, the stresses in the clay approach the ultimate strength of the clay so that movements occur which are often of considerable magnitude. If the ultimate strength of the clay has been determined in the laboratory by unconfined compression tests, the designer will know at what stress he can expect the clay to fail completely. If the laboratory determines the elastic limit or the yield point of the clay by means of the ring-shear test, the designer will be forced to use some kind of correction factor such as the "overload ratio" described by Professor Housel.

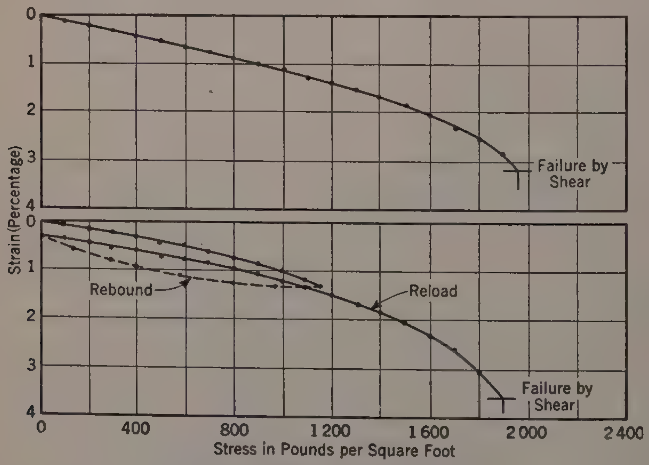

Fig. 50

Consideration of the significance of the overload ratio leads to the conclusion that it must depend on several factors which can vary over a wide range. In the first place, the numerical value of the overload ratio must depend on the ratio between the stress at which the clay fails and the stress at which it starts to flow. In the second place, the numerical value of the overload ratio must depend on the effects of progressive failure in the soil mass in the field.

As to the ratio between failure stress and yield stress, this can be practically unity but it might be as great as 4 or 5 if not greater. For example, Fig. 50 shows the results of several unconfined compression tests on undisturbed samples of the very fine grained clay that underlies Mexico City, Mexico. This soil had a natural water content of about 400%; a plastic limit of about 125%; and a liquid limit of about 500%. As can be seen from Fig. 50, the stress-strain curves are practically straight lines all the way out to the failure load. In spite of its plastic properties, this clay behaves almost like a brittle material. There is no yield point until the failure load is reached, so that the ratio between failure stress and yield stress is practically unity. On the other hand, Fig. 49 indicates that for the Indiana Harbor clay, the failure stress determined by the unconfined compression test is about five times the yield stress determined by the ring-shear test. It is entirely reasonable to expect that the ratio between failure stress and yield stress might be even greater than 5 in some cases. Accordingly, a designer, basing earthwork computations on the yield value determined in the ring-shear test, would have a difficult problem in choosing a proper value for the overload ratio. The tendency, almost certainly, would be to stay on the conservative side.

The effect of progressive failure in the soil mass in the field can also vary over a considerable range. The term "progressive failure" refers to the fact that the soil does not necessarily fail everywhere at the same time. In some cases, progressive failure occurs because the stresses are much higher in some parts of the clay bed than they are in others. For example, if a square bearing block is used to make a load test on plastic clay, the soil will fail at the corners and edges of the plate before it fails under the center of the plate. In other cases, progressive failure occurs because the shearing strength of the clay varies in different parts of the clay bed. In such a case, the weaker layers might fail first, leaving the stronger layers to carry more than their share of the load. It is very probable that progressive failure is often caused by a combination of these two factors. In any case, the effect of progressive failure would have to be taken into account in the choice of a numerical value for the overload ratio. Here again, the tendency on the part of the designer would be to stay as far as possible on the conservative side because he is dealing with failure stresses whereas the laboratory tests have determined the yield stresses.

From considerations of this kind, the writer has come to the conclusion that the unconfined compression test procedure represents a more direct approach to problems of this type than does the ring-shear test procedure. The construction of tunnels, open cuts, or large cofferdams in plastic clay beds always involves some flow of the clay toward the excavation. In other words, the clay is stressed well beyond its yield value, and in some parts of the clay bed the stresses might reach the failure value due to the effect of progressive failure. Accordingly, it seems to the writer that it is more logical to determine the failure values in the laboratory and to work with them rather than to determine the yield values and then to increase them by means of an overload ratio.

There is one point of considerable importance which is referred to by each of the Symposium authors. Apparently, they all agree that the shearing resistance in natural deposits of soft plastic clay is due to cohesion only. In other words, the angle of internal friction is zero and the normal pressure on a given plane does not influence the shearing resistance on that plane. This is a point that should be kept in mind by designers making earthwork computations involving natural beds of soft plastic clay.

D. M. Burmister,[30] Assoc. M. Am. Soc. C. E.[30a]—This Symposium is an outstanding contribution to the knowledge of the forces and physical factors involved in tunnel and open-cut construction in plastic clays. The authors are to be congratulated for the presentation of this valuable information to the profession. The importance of their investigation is threefold: (1) A rational scientific approach is offered for the investigation and the practical solution of such problems; (2) usable criteria are suggested for safe and economical design and construction; and (3) control criteria and procedures to be used during construction are developed to meet and anticipate changes in subsurface conditions for the worse and to minimize settlement due to tunneling and open-cut operations.

The writer was impressed greatly by the comprehensive program of soil investigations conducted on a time schedule that permitted the use of the information for construction purposes. This was obviously not a research problem, but a practical problem of soil mechanics on a mass-production basis. The construction experience itself is evidence of the thoroughness and the efficiency with which this program was planned. It is interesting to note that reliance was placed primarily on the results of two simple soil tests—moisture content and unconfined compression—which had a direct bearing on the problem in hand. The method of presenting and interpreting the data is especially noteworthy because of its value in clearly visualizing subsurface conditions and in indicating the nature of the difficulties involved.

Professor Terzaghi has made a number of statements of experience in connection with the construction of the Chicago subway which are of great importance. First, he states that when dealing with irregularly stratified water-laid deposits, and also the ice-laid deposits of the Chicago region, the average soil properties do not change, in a horizontal direction, nearly as erratically as the individual records would lead one to expect. Thus, considerable reliance can be placed on the data regarding subsurface conditions obtained from quite widely spaced borings, as commonly used. The greatest uncertainty lies in the possibility of missing some abrupt change for the worse in the thickness of the deposit or in stratification. Analyses of foundation conditions for a number of important projects, covering quite large areas, by the writer have borne out similar conclusions, including the consolidation properties of clays and critical density-pressure relations for deposits of fine sands.

In a second conclusion of great consequence, Professor Terzaghi states that, on account of the mechanical properties of undisturbed clays, a very small yield suffices to develop all the strength the clay has. In discussing the lateral pressures in the Chicago clays, Mr. Peck states that a lateral yield of slightly less than 0.25% of the height of the cut is all that is necessary to reduce the earth pressure to the active value. This is in contrast to the opinion commonly held that opportunity for lateral expansion sufficient to develop the major part of the shearing strength of soft clays is never realized. Obviously, the yield required will depend on the strength of the clay itself and on the stress-strain properties. The zone in which the lateral yield must take place may be considerably more restricted or limited in extent than formerly believed. In the closing discussions, it would be of great value to include relations, if consistent ones were found, between the strain at the maximum unconfined compressive strength and the measured distortion in the tunnel section, the "squeeze" into the tunnel, and the lateral yield in the open-cut section.

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[30] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[30a] Received by the Secretary March 24, 1943.

A third conclusion of great importance is that the medium soft clay acts as if the angle of friction were equal to zero and that the shearing strength did not increase with the depth of overburden. Hence, the unconfined compressive strength was considered to be more indicative of the shearing strength of the clay. Professor Terzaghi states that he has not seen any evidence of the existence of a direct relation between shearing strength and depth of overburden in soft clay deposits. This same phenomenon exists in the softer Detroit clay to an even more marked extent, as discussed by Professor Housel and as shown in Fig. 39. The writer has discussed a similar experience[31] in connection with the investigations of the subsurface conditions of the very soft deposit of organic silty clay at the Flushing Meadow Park site for the New York (N. Y.) World's Fair. The moisture contents, shearing strengths, and preconsolidation pressures were constant throughout almost the full depth of the deposit, some 40 to 60 ft in thickness below the surface root mat, whereas in a natural state of complete consolidation one would expect that the water content would decrease and the preconsolidation pressure, and hence the shearing strength, would increase almost directly with the depth in the deposit. Such a radical departure may be the normal natural state of consolidation under very slow geologic loading; or it may be explained on the basis of the presence of artesian water pressure in the sand strata beneath the deposit. Such a condition, with an infinitesimal flow upward, would induce permanent hydrostatic stresses in the pore water, which would limit consolidation so that a certain proportion of the overburden pressure at every depth would be carried permanently by pore water pressure. However, the shearing strength, in any case, was found to be substantially directly proportional to the preconsolidation pressure evaluated from consolidation tests, whether it is less than, equal to, or greater than the existing overburden pressures, which vary with the geologic load history of the deposit. This preconsolidation pressure represents the effective stress acting on the grain structure of the clay. In a closed system (which is always the case in a large mass of relatively impermeable clay), any increase in stresses from any cause above the preconsolidation pressure is carried entirely by excess hydrostatic stresses in the pore water. This is a consequence of the characteristic phenomena in clays—a tendency toward a decrease in volume under shearing stress—which always accompanies shearing deformations. Hence, at constant volume shearing, the clay behaves as though the angle of friction equaled zero, and only the shearing strength due to the pre-existing effective stresses can be mobilized. Because of this phenomenon, the unconfined compression test on reasonably undisturbed samples yields the most representative and reliable measure of the natural shearing strength of the clay, as stated by Professor Terzaghi. The writer prefers to think of the maximum shearing stress for such a condition as being determined by one half the difference of the principal stresses—$S_{\max} = \frac{1}{2}(p_1 - p_3)$. Since the applied lateral pressure p_3 is zero in the unconfined compression test, the maximum shearing strength is equal to one half of the maximum applied vertical stress, which is the compressive strength of the clay. In obtaining representative values from unconfined compression tests, it has been the writer's experience that the shearing strength obtained by testing a specimen having the full diameter of the undisturbed sample may be only 75% of that obtained from a prism carefully cut from the central part of the sample with a maximum width, about one half of the diameter of the undisturbed sample. Also, it has been his experience that direct quick shear tests yield values which may be 90% or less than those obtained from unconfined compression tests on specimens cut from an immediately adjacent part of the sample. This is believed to be a consequence of the progressive nature of the failure conditions inherent in the direct shear test method. Also, this may account for the fact noted by Professor Housel that, for a perfect check between measured and estimated pressures on the tunnel, the shearing resistance should be 12% greater than the yield value that was found experimentally by ring-shear laboratory tests.

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[31] "Laboratory Investigations of Soils at Flushing Meadow Park," by Donald M. Burmister, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 187.

The outstanding achievement of this Symposium is the development of rational methods of analysis, design, and control of construction procedures for tunnel linings and sheeting and bracing in open cuts in plastic clay, which are in such close agreement with actual measurements. The most important practical aspects for future construction are the measurements of: (*a*) The distortions and pressures in the tunnel lining in the Chicago clays and the settlement of street surfaces; (*b*) the magnitude and distribution of pressures in open cuts and the yield in the bracing system permissible to limit adjacent surface settlement; and (*c*) the distribution of pressures in the tunnel lining in the Detroit clay, the variation with time, and the time required for equilibrium condition to be attained.

In order to make these measurements of greater practical value for future construction, the properties of the Chicago and Detroit clays should be identified more positively. It is suggested that diagrams showing the range of soil properties and conditions should be included, if fairly consistent relations exist, for example: (1) Maximum unconfined compressive strength or shearing strength versus moisture content, relative density—$D\,d = \frac{W_L - W_N}{W_L - W_P}\,100\%$, and corresponding strain at maximum strength; (2) maximum shearing strength and corresponding strain versus the measured distortion in the tunnel lining, the squeeze into the tunnel, the pressure on the tunnel lining, and the ratio between the vertical and horizontal pressures observed; and (3) maximum shearing strength and corresponding strains versus the measured pressures and yield in the bracing system in open cuts.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING

Discussion

BY MESSRS. CLARENCE S. JARVIS, IVAN E. HOUK, W. G. HOYT, AND L. R. BEARD

CLARENCE S. JARVIS,[4] M. AM. SOC. C. E.[4a]—This timely, concise report on "relative successes and failures" has a special appeal because of its clarity of concept, its modesty of claims, and its wealth of significant data in their proper relationships. The final paragraph preceding "Acknowledgments" deserves particular emphasis, which may be afforded by an illustration from the writer's personal experience.

In connection with a paper on the characteristics of flood flow (23),[4b] the insertion of a few brief paragraphs on rainfall phenomena, particularly in their relation to floods, was found desirable; and several months of study under the local Weather Bureau officials were required before that portion of the text attained its final form. Incidentally, that course of intensive study opened the way to the writing of a companion paper (24), for which the nucleus was prepared and diverted from the earlier paper. Fully five years were needed to encompass the first phase of that project, namely, the assembly and summarization of representative precipitation data from stations throughout the world. Basic principles and interesting relationships were set forth as a preliminary to solving some of the outstanding problems involving rainfall and resulting runoff. The data proved to be so abundant and varied, and the field for investigation so broad, that it seemed best to publish the assembled records with such brief discussion of fundamentals as had been prepared and to invite cooperation from all hydrologists, engineers, and interested scientists or research workers in the interpretation of facts brought forth and in the extension of the hydrologic field.

NOTE.—This paper by Merrill Bernard, M. Am. Soc. C. E., was published in January, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Edgar Dow Gilman, M. Am. Soc. C. E.

[4] Hydr. Engr., With Office, Chf. of Engrs., Washington, D. C.

[4a] Received by the Secretary March 1, 1943.

[4b] Numerals in parentheses, thus: (23), refer to corresponding items in the Bibliography (see Appendix 1) at the end of the paper, and at the end of discussion in this issue.

In the course of writing chapters for recent volumes, all dealing with some phase of hydrology, the writer has observed that the progress is so rapid that extensive study is required for one to avoid becoming out of date.

After reviewing the most notable recent advances in practical hydrology and meteorology now largely associated with the demands, as well as the facilities, afforded by aeronautics for obtaining three-dimensional weather data, one is impressed with the insignificance of any new ideas he has brought forth, in comparison with the important problems awaiting and inviting solution. Frankly, within the writer's experience, some of the methodology and findings that rated highest according to his judgment were promptly relegated to the rôles of "incomplete forward passes." Probably the surest way to have new procedures adopted for general use is to have them devised and announced by responsible, thoroughly organized agencies. This leads inevitably to the conclusion that such official representatives should continue to set the pace and to direct the vanguard in notable advances. Unofficial utterances possibly may attain the importance of "aerial reconnaissance," or even desultory "bombing" of a hostile, unknown area. Complete mastery of the situation will continue to depend on the organized efforts of "ground troops" and their occupation in force.

Although accepting the authoritative data and their analysis as presented in this paper, the writer submits herewith some qualifying notes and queries, which seem to invite further study:

1. Solar radiation may contribute directly to the heating of the atmosphere through which it passes—probably somewhat less than 10% for clear, dry air when the sun is near the zenith, but correspondingly more as the rays become more inclined. Examples of equivalent transparent masses are 34 ft of clear, placid water or 14 ft of the best grade of glass. With the formation or the gathering of cloud masses, the reflecting and radiation surface is elevated high above the ground and applied to the heating of a lesser mass of atmosphere, in a rarefied condition, resulting in pronounced thermal changes and gravitational instability therein.

Is it not quite probable that much of the turbulence and convective movement results directly from the local effects of solar energy applied to the upper surface of the cloud masses? Such heating should dispel the upper fringe of cloud forms, and the induced buoyancy would give an upward component to the air column's movement.

2. The author and other writers assume that contained in the atmospheric column covering the globe is a single inch of water, on an average, with appropriate excess in torrid regions and deficiency near the poles. If this is correct, it seems inconsistent to assume a depth of only 0.88 in. for the precipitable moisture on the date of heaviest rainfall, during the storm which most nearly represents the maximum possible. It is of record that more than 1 in. fell at Cincinnati, Ohio, in the hour extending somewhat beyond midnight of March 25–26, 1913; and considerable areas recorded as high as 6 in. in 24 hr. It would have required more horizontal wind movement than has been recorded to account for the seven complete displacements and renewals of air columns over those extensive areas of high rainfall; and those same areas would have

been flanked by regions of deficient rainfall, if the air masses had been "wrung dry" in the convective cells.

On the contrary, the 1,100 sq miles of area with precipitation depth 6 in. or greater were flanked by areas of moderate rainfall depth; 12,000 sq miles with 4 in. or more, and some 20,000 sq miles with 3 in. or more, and much greater areas for the 2-in. and 1-in. isohyetals. Assuming that the width of incoming tropical airstream was 200 miles, then the movement required for extracting the average depth of 4.7 in. within the 4-in. isohyetal is expressed by: Horizontal wind velocity in miles per hour, $V = \frac{4.7 \times 60}{0.88 \times 24} = 13.3.$

TABLE 3.—STORM OF MARCH 24 TO 27, 1913, AS RECORDED AT U. S. WEATHER BUREAU STATIONS IN CINCINNATI (C) AND DAYTON (D), OHIO

March	Temperature, in Degrees Fahrenheit						Wind Velocity, in Miles per Hour				Precipitation (Inches of Depth)	
	Maximum		Mean		Minimum		Maximum		Mean			
	C	*D*	*C*	*D*	*C*	*D*	*C*	*D*	*C*	*D*	*C*	*D*
24	70	64	62	58	55	51	18 W	50 S	....	18	2.21	2.95
25	70	59	63	54	56	48	26 SW	46 S	....	....	4.15	2.27
26	57	56	47	44	37	32	17 NW	....	....	....	1.11	....
27	37	32	32	28	27	23	22 W	....	....	....	trace	1.90 (estimated)

To supply the larger areas with lesser depths included in this particular storm pattern, much greater wind velocities would have been required. Assumption of greater precipitable depths of moisture would reduce the corresponding wind movement, at least to stay within the maximum velocities observed at the Cincinnati station, as recorded in Table 3.

Quoting from a previous paper (24*a*):

> "The condensation of 1 in. of rain releases the equivalent of 2 h.p.-hours of energy per sq.ft. of surface, or nearly 87 000 h.p.-hours per acre. If applied exclusively toward heating the first mile in height of air column above sea level, the resultant increase of temperature would be 30° Fahr., or more, depending on the density and humidity; and this thermal change would cause an expansion ranging from 6 to more than 10 per cent. The resulting buoyancy would be of the same order as that possessed by ice or logs floating in water."

There is no more assurance of confining the released energy to the heating of the lower mile of the atmospheric column than of applying similar heat units solely to the melting of snow cover as the vapor condenses upon its surface. A considerable time factor is required to complete the transfer of the heat units—some portion to the condensed moisture, some to the snow, and the remainder to the surrounding atmosphere. Furthermore, the natural surface runoff on perceptible slopes may limit the time of contact for the condensed moisture and the snow surface, thus preventing complete transfer of the released energy. During a cold spell following a heavy snowstorm in Washington, D. C., attempts were made to remove some of the cluttering snow heaps at intersections by the application of blowtorch flames to tunnel through the compact mass. The slight trickles of water reaching the gutters and the slow progress in snow removal plainly demonstrated that heat was being dissipated into the surrounding atmosphere instead of into the packed snow.

Inasmuch as the first volume of daily *Climatological Data* began in January, 1914, before much progress had been made in regular observations aloft, it may be well to present some of the available records, pertaining to the storm of March 24 to 27, 1913, in Ohio (see Table 3).

In the absence of direct observations as to upper air conditions, only estimates and assumptions are available. Nevertheless, condensation and precipitation were proceeding vigorously at the prevailing temperatures as observed at U. S. Weather Bureau stations; therefore, the actual dew point for some of the air masses involved in the turbulence might have been somewhat higher than the station temperatures. At Kew, near London, England, it was observed that the normal variation of dew point under foggy conditions was nearly half the diurnal variation in station temperature, thus amounting to some 13° C or 23.4° F (17*a*). W. Napier Shaw has presented tabular data (25) showing the water-vapor capacity of saturated air up to 30,000 ft above sea level as more than 0.73 in. in depth when the temperature at the base of the column is 50° F. Field observations at St. Louis, Mo., with a maximum observed temperature of 77° F, gave the atmospheric moisture capacity at saturation as 2.28 in., of which 1.67 in. was subject to precipitation; and other stations recorded moisture capacities of 2.60 and 2.92 in., respectively, for temperatures of 77° and 73°, with 1.67 and 1.41 in. subject to precipitation (26).

3. Whether the temperature gradient was subjected to inversions, increasing the moisture capacity, or to other irregularities resulting from the general turbulence, the heavy precipitation and the corresponding release of energy from the condensing vapor was not recorded, as upper air data were not observed. No one can say authoritatively and convincingly, therefore, that the total moisture content of the air column was only 1.40 in. nor that the precipitable moisture at the beginning of the storm was only 0.88 in. or double that amount. Of vastly more importance are the foresight, determination, and courage displayed both in the establishment and the maintenance of the Hydrometeorological Division and in its guidance toward the solution of vital problems affecting the national security and economy.

After all, the rainfall-runoff relationship is a simple one, despite the complicated, elusive, and partly indeterminate factors involved. One of the important keys to its solution, the "unit hydrograph" (27), written following the November, 1927, floods in New England, seems destined to stand as a monument to organized research. This clear concept, by L. K. Sherman, M. Am. Soc. C. E., was developed further by Mr. Bernard's keen perception and adroit adaptations—together with the determined efforts of the U. S. Geological Survey; the State of Pennsylvania; the Corps of Engineers, U. S. Army; the U. S. Weather Bureau; the Soil Conservation Service; and other federal or local agencies—to provide a safe and logical basis for analysis, forecasts, and design related to floods and plans for their control. Naturally, important contributions have come from the vanguard among consultants, such as W. W. Horner, M. Am. Soc. C. E., Robert E. Horton, M. Am. Soc. C. E., and Horace W. Wood, Jr., M. Am. Soc. C. E. Mr. Wood's work, "Flood Flow on Missouri Streams" (28), gives an unusually clear résumé of methods now in vogue, including empirical and other formulas together with the "unit hydrograph" in their appropriate settings.

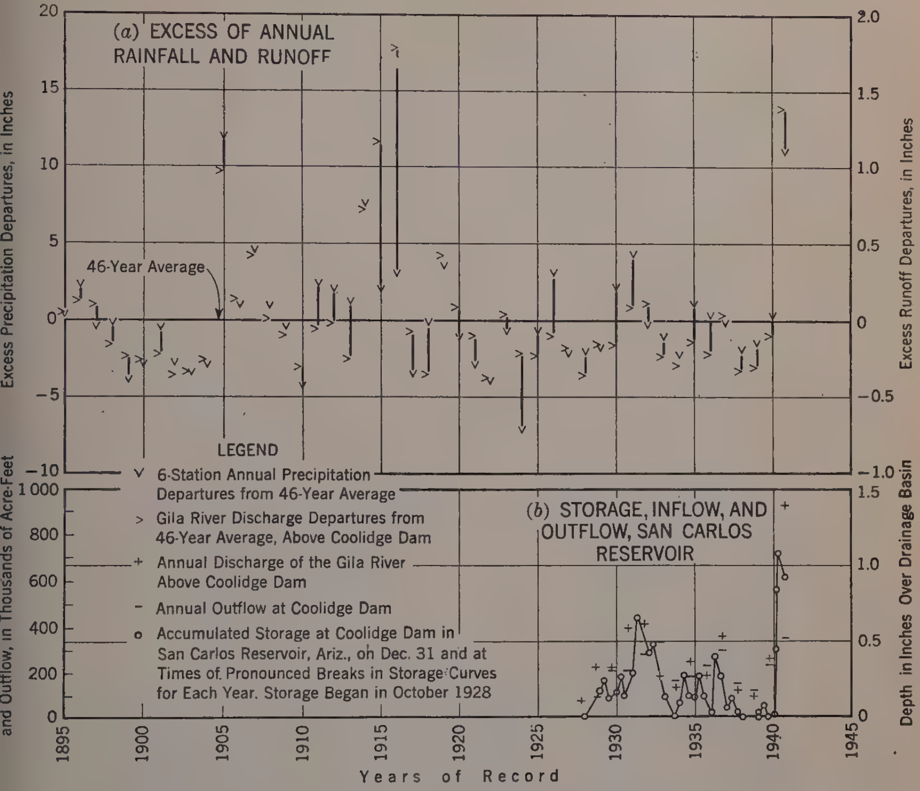

FIG. 21.—GILA RIVER BASIN, AS OBSERVED AT COOLIDGE DAM, ARIZONA; ANNUAL PRECIPITATION AND RUNOFF EXCESS COMPARED WITH 46-YR AVERAGES

In spite of the varied topography and hydrologic as well as meteorological characteristics of the semiarid Southwest, the writer has found some very consistent relationships between rainfall and runoff excess or deficiency, year by year, from 1895 to 1941, inclusive.

The 46-yr average values are as follows:

Description	Gila River basin	Rio Grande basin
Precipitation, in inches	11.80	12.83
Runoff:		
Inches of depth	0.492	0.840
Cubic feet per second	968	1,500
Thousands of acre-feet per year	339	1,084
Area:		
Square miles	12,880	24,200
Thousand acres	8,243	15,488

The foregoing yearly means of rainfall depths at six stations on the Gila River watershed above Coolidge Dam, Arizona (Fig. 21), and of the nine stations on the Rio Grande watershed above Elephant Butte, N. Mex. (Fig. 22), were subtracted from the respective 46-yr means for those same station groups, to give the excess or deficiency for each year. Corresponding departures from annual mean runoff depths per year throughout the same record periods, plotted on a scale 10 times as great, show fair agreement generally, with several of the years recording nearly coincident plottings. Likewise,

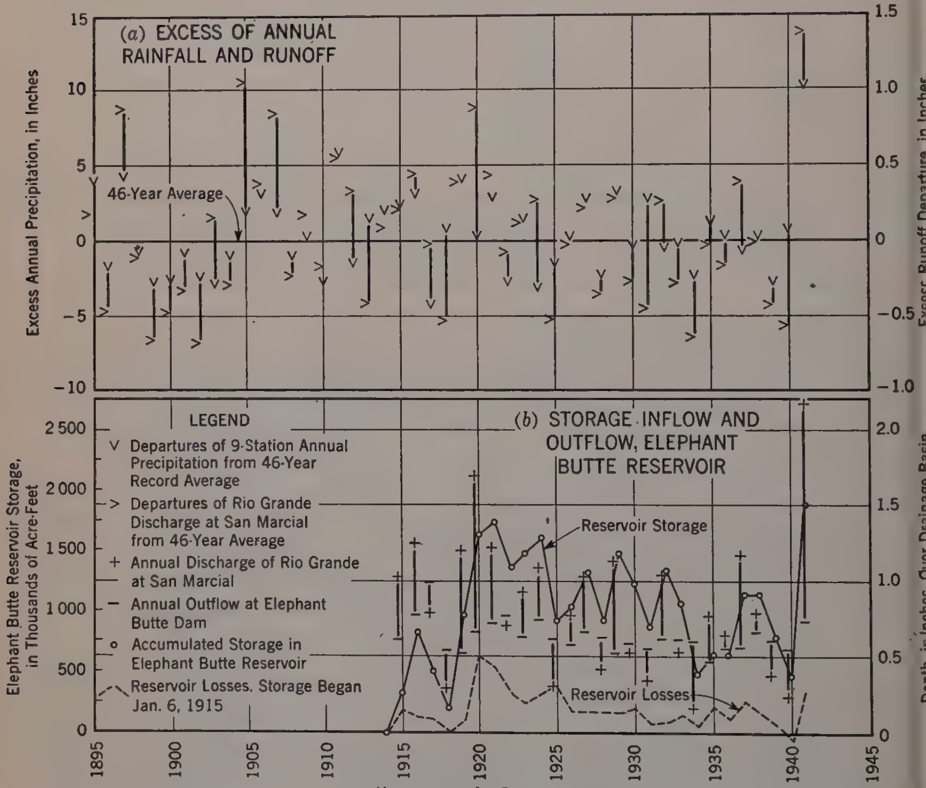

FIG. 22.—RIO GRANDE BASIN, AS OBSERVED AT SAN MARCIAL DAM, NEW MEXICO; ANNUAL PRECIPITATION AND RUNOFF EXCESS COMPARED WITH 46-YR AVERAGES

the cumulative effect of a series of relatively wet years, such as 1918 to 1920 on the Rio Grande and 1905 to 1907 or 1911 to 1916 on the Gila River, and the subnormal rainfall generally from 1932 to 1940 on both watersheds, culminated in great departures from normal runoff and associated storage volumes, either in underground or in major surface reservoirs.

In 1941, with rainfall nearly double the normal, runoff was recorded representing nearly 4 times the mean for the Gila River, and more nearly 2.7 times the mean for the Rio Grande at Elephant Butte (where the excess was 1.7 times the mean).

Sufficient data were available to determine the annual losses in Elephant Butte reservoir, with a mean of about 200,000 acre-ft and a maximum of 648,000 acre-ft in 1920 (during the initial filling of the higher contours which required heavy contributions to bank storage). During 1918, the reservoir losses were nearly balanced by return flow from bank storage; and, during the critically dry season of 1940, the return flow not only equaled the storage losses, but also provided a dividend of 48,000 acre-ft for use in the lower valley.

A similar approach would disclose interrelationships between either monthly or daily rainfall and runoff, provided all of the principal factors, such as antecedent rainfall and temperature, and their effect upon soil moisture, are given due consideration. Similar relationships for the time, distance, and volumetric quantities, which affect both peak and total discharge at successive stations along the river channel, would be found.

Neither the microscopic approach, through rainfall intensities per hour or minor fraction thereof, nor the telescopic approach, through record means and departures therefrom is capable of answering all the vital questions pertaining to water supply, flood forecasting, and best schedule for release of reservoir storage. Both of those approaches, and others occupying intermediate ground, should be regarded as independent witnesses contributing evidence on the problems involved. The author has done much toward integrating the available records as well as the logically derived implications, and thus has completed one of the "forward passes" previously attempted by the writer (23)(24) before such a wealth of data, methodology, and coordinated teamwork was at hand.

IVAN E. HOUK,[5] M. AM. SOC. C. E.[5a]—Estimates of maximum possible flood flows must often be made for use in designing spillways and other hydraulic structures, as well as for use in planning major flood control projects. The author's comprehensive discussion of meteorological problems involved in such estimates shows that real progress is now being made on this important phase of hydrologic work. The accompanying data regarding maximum possible storms and their relation to maximum possible floods should prove valuable to practicing hydraulic engineers, to flood control specialists, and to all scientists actively engaged in meteorological and hydrological researches.

Naturally, the occurrence of the maximum possible storm on a given drainage area does not necessarily mean the occurrence of the maximum possible flood, even though the duration of the storm rainfall may equal the time of concentration for the given area. This is especially true for relatively large drainage basins in temperate zones, where conditions of snow cover and surface soil moisture content constitute important influences on rates of flood runoff. It gradually becomes less certain as the size of the basin decreases. In the case of a comparatively small area subject to intense rainfall of the thunderstorm or cloudburst type, the maximum possible flood probably is caused by the maximum possible storm, since the rate of infiltration (even under optimum conditions) usually is only a small proportion of the rate of cloudburst precipitation.

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[5a] Received by the Secretary February 23, 1943.

The world's records of most intense point rainfall shown in Fig. 7 suggest the question as to whether or not they represent maximum possible rates of rainfall. The question is primarily of academic interest, since it probably never would be economically advisable to design bridges, culverts, sewers, spillways, or other hydraulic structures for the intense rates of runoff which such precipitations would produce. Probably the points shown on the diagram represent rates somewhere between 90% and 100% of maximum possible precipitation rates.

The discussion of the theoretical determination of storm rainfall constitutes an interesting introduction to a complicated meteorological problem. Although some items involved in the suggested method of calculation are not at present susceptible of accurate evaluation, the general method of attack seems logical and the continuance of such studies should lead eventually to a better general understanding of the problem. The application of Eq. 2 to some actual storm records and the publication of the results, including values of K, v, and W_E, would constitute a valuable addition to the paper.

W. G. HOYT,[6] M. AM. SOC. C. E.[6a]—In summarizing the progress made in one phase of hydrometeorology during the ten years since his earlier study in 1934 (29), Mr. Bernard has performed an excellent service to the engineering profession. In 1934 no attempt was made to answer the question as to whether or not the transposed storm was a "limiting" storm. The principal question raised at that time by the advising committee of hydrologic experts was whether or not, with reasonable likelihood, the great storms could have shifted slightly within a region, to center in a critical position on the particular basin of interest. As a result, however, of the research studies and observations described by Mr. Bernard, the hydraulic engineer of 1943 not only can transpose storms with reasonable surety but can determine, at least theoretically, with the meteorologic information now at his disposal, the potential limits of storm precipitation.

Credit certainly should accrue to the personnel of the various government agencies who have worked on the interrelated hydraulic and meteorologic problems and to the Corps of Engineers who in many instances made funds available for such studies. Not to be forgotten is the engineering staff of the Miami Conservancy District who early organized storm data for flood studies.

On June 15, 1932, a special Society Committee on Meteorologic Data submitted a progress report (30) in which they concluded among other things that, as of 1931, "Research in meteorology by the [Weather] Bureau, particularly along the lines which are used by the Engineering Profession, has not kept pace with the growing importance or utilization of this type of data, nor with research in other lines of science, either pure or applied." One reasonably might conclude from the evidence of Mr. Bernard's paper that, as of 1943, the research in meteorology—especially as it relates to the engineer in flood problems—is outstanding.

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[6a] Received by the Secretary March 11, 1943.

L. R. BEARD,[7] JUN. AM. SOC. C. E.[7a]—Considerable progress has been made during the past several years in developing the science of hydrology. Perhaps the most difficult and most interesting development concerns the relation of rainfall to flood flow. This newer feature promises to rival that development in both difficulty and interest.

Because of the value of resolving a variable into its more fundamental components, it is advantageous to deal with meteorological factors rather than with observed precipitation. A disadvantage of dealing with meteorological factors is that, as Mr. Bernard states (see heading: "Principles of Storm Transposition"), there is available "a scanty 8 years of reasonably adequate upper-air data." These data are of utmost importance in the quantitative determination of precipitation.

In replacing a general function such as flood flow by its several component functions—infiltration, absorption, evaporation, return flow, transpiration, snow melt, detention, siltation, air moisture, wind velocity, frontal activity, convective activity, cyclonic activity, etc.—it is primarily necessary to relate the various component functions properly. However, if advantage is to be gained by this replacement, the probable error of the result obtained by using the component functions should be less than the probable error of the result obtained by using the general function. With the probable error of the result of each component function known, the probable error of the combined result can be computed by the equation (31):

$$R = \left[\sum_{j=1}^{j=m} \left(\frac{\partial Z}{\partial z_j} \right)^2 (r_j)^2 \right]^{0.5} \dots\dots\dots\dots (3)$$

in which there are m component functions, and in which: R = probable error of the combined result; Z = the combined result expressed in terms of the component results; z = a component result; and r = probable error of a component result. The relation of runoff to the component factors is not often a simple differentiable function, and much simplification would be necessary in order to apply Eq. 3 mathematically. However, it should be noted that the probable error of the sum or difference of m quantities is equal to

$$r_m = \left[\sum_{j=1}^{j=m} (r_j)^2 \right]^{0.5} \dots\dots\dots\dots (4a)$$

and the probable error of the product of two numbers is equal to

$$r = [(z_1)^2 (r_2)^2 + (z_2)^2 (r_1)^2]^{0.5} \dots\dots\dots\dots (4b)$$

The important point to be made is that the possibility of evaluating one factor very accurately should not be interpreted to mean that the combined result is necessarily evaluated accurately. If the maximum possible rainfall could be evaluated with great accuracy, then the probable error of the maximum possible flood derived therefrom would be slightly greater, on a percentage basis, than the probable error of the maximum possible over-all runoff coefficient. In general, the function "runoff coefficient" expressed in terms of infiltration, snow melt, detention, etc., is less reliable than the "runoff function," since it is derived from the latter. However, the runoff-coefficient function may converge more rapidly than does the runoff function, for which reason it is questionable whether the maximum possible runoff can be determined more accurately from rainfall than from runoff. The method by which maximum accuracy is obtained must be selected independently for each case.

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[7a] Received by the Secretary March 16, 1943.

In the determination of a maximum possible value of a function such as wind velocity, the procedure is not entirely mathematical. In general, such a value is merely considerably greater than any observed value (this is substantiated by the fact that the value is referred to as "maximum probable" by many). It is true that, in most cases, one scarcely can imagine a larger value. However, one hardly could imagine the occurrence of the Republican River flood of 1935 prior to that time. When the determination of the many factors (component functions) is made by "engineering judgment," a suggestion that the accuracy of the combination be investigated may be looked upon as a bit farfetched. However, the accuracy may be evaluated in the same manner as the quantity with the same justification.

There is one other point to be considered which is somewhat related to the accuracy discussion. Spillway and other designs sometimes are based on the maximum possible or maximum probable flood. In these cases, maximum possible values necessarily are evaluated as low as judgment allows (in order that construction costs will not be prohibitive). Remote or hazy possibilities are discounted. As a result, since the value cannot be strictly designated as maximum possible, it often is called maximum probable. The best that can be said for such a quantity is that it is perhaps a very large quantity and that it is the result of a decision making a design possible. Such a decision might not have been made if a more definable quantity were demanded. In other words, it "gets the job done." The worst that might be said for such a quantity is that its use as a maximum possible value would cause the following:

(*a*) The engineer would use the value as a basis for his calculations and his judgment and, consequently, for his decisions, and therefore would not have a sound basis for such decisions; and

(*b*) The economic calculations would be in error because the possibility of a disaster resulting from an occurrence in excess of the maximum probable occurrence is neglected or overlooked.

One example of a remote possibility which is ordinarily neglected is the extreme condition of snow melt. Partly melted snow could exist as slush in such a state that a small amount of heat would release tremendous quantities of water. Under these conditions, the maximum possible runoff volume virtually would be unlimited.

In designating a region as being meteorologically homogeneous, certain exceptions must be admitted. The "western side of the Pacific Coast Ranges," for example, can be considered meteorologically homogeneous for air-mass qualities, upper wind velocities, and many other qualities, with certain toleration. However, one rightfully cannot transpose the rainfall data shown in Table 1(*j*) to any part of the region. This is because the areas in which these excessive rainfall amounts occurred are areas of normally high rainfall and cannot be considered to be on a par with other areas in the region. Opids Camp, Calif., has a normal seasonal rainfall of almost three times that of Los Angeles, Calif., which is about 15 miles distant. Almost invariably the rainfall at Opids Camp is two to three times that at Los Angeles. In the mountains north and east of Los Angeles, the area which could be said to have reasonably constant rainfall characteristics would be limited to a few square miles in extent. There are undoubtedly many other areas in the United States where simple storm transposition would be impracticable and where the condition of meteorological homogeneity is restricted.

The convective cell of Fig. 17 is the type of phenomenon which produces cloudburst or thunderstorm precipitation. The height of such a cell normally is limited by the depth of an instability layer and always is limited by the height of the tropopause. Experiments with fluids which are not exactly analogous to air have shown that the horizontal diameter of such a cell is on the order of three to four times the depth of the cell. As Fig. 17 shows only the rising half of the cell's cross section, the drawing of the descending (outside) part of the cell would complete a cell whose diameter, in this case, is about two times the depth. Condensation would occur in the part of the cell shown (ascending air), and rainfall therefore would occur below and near this part. Only very light rainfall, if any, would occur below the descending part of the cell.

The type of rainfall which is steady and almost uniformly distributed over large areas generally is associated with a cyclone and, often, a frontal system. The cyclone, which may extend far into the stratosphere, is a result of dynamic instability and cannot be classed as a convective cell, which is a result of thermal instability. Often, however, thermal instability is present in the vicinity of a cyclone, in which case both types of rainfall are superimposed.

Numerous other complicated factors should be considered before a meteorological analysis can be attempted for any area. Some admirable work has been done in this line by J. Bjerknes, consulting meteorologist, and by the Pacific Division Office of the U. S. Engineer Department at San Francisco, Calif. This work should be of considerable value for the formulation of a procedure of analysis.

Bibliography.—

(17a) **"Weather Analysis and Forecasting,"** by S. Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 124, Fig. 68.

(23) **"Flood Flow Characteristics,"** by C. S. Jarvis, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), pp. 985–1032.

(24) **"Rainfall Characteristics and Their Relation to Soils and Runoff,"** by C. S. Jarvis, *ibid.*, Vol. 95 (1931), pp. 379–423. (*a*) p. 391.

(25) **"Weather Forecasting,"** by W. Napier Shaw, Constable & Co., Ltd., London, England, 1941, p. 220.

(26) Dictionary of Applied Physical Science, Vol. III, p. 61.

(27) "The Unit Hydrograph and Its Application," by L. K. Sherman, *Journal*, Associated State Eng. Societies, April, 1941, p. 4.

(28) "Flood Flow on Missouri Streams," *Bulletin No. 30*, Univ. of Missouri Eng. Series, 1942.

(29) "Unit Hydrograph Method and Storm Transposition in Flood Problems Relating to Great Storms in Eastern and Central United States," by Merrill Bernard, *Water Supply Paper No. 772*, U. S. Geological Survey, 1936.

(30) "Meteorological Data," *Proceedings*, Am. Soc. C. E., Vol. LIX (1933), pp. 153–182.

(31) "Handbook of Engineering Fundamentals," by Ovid W. Eshbach, John Wiley & Sons, Inc., New York, N. Y., 1936, p. 2–127.

Corrections for *Transactions:* In January, *Proceedings*, page 107, delete lines 23 to 27, with Fig. 2, and substitute:

"* * * Wakefield, Kans., that the discharge in the Republican River could have greatly exceeded 70,000 cu ft per sec. The maximum flood flow of record had increased from 37,500 to 47,500 cu ft per sec in 1903, and to 70,000 cu ft per sec in 1915, but for the next twenty years the river failed to reach or exceed the 1915 crest discharge. In May, 1935, the Republican River Basin was visited by a storm which produced a flood peak estimated at 180,000 cu ft per sec. The prevailing maximum-flood discharge, * * *."

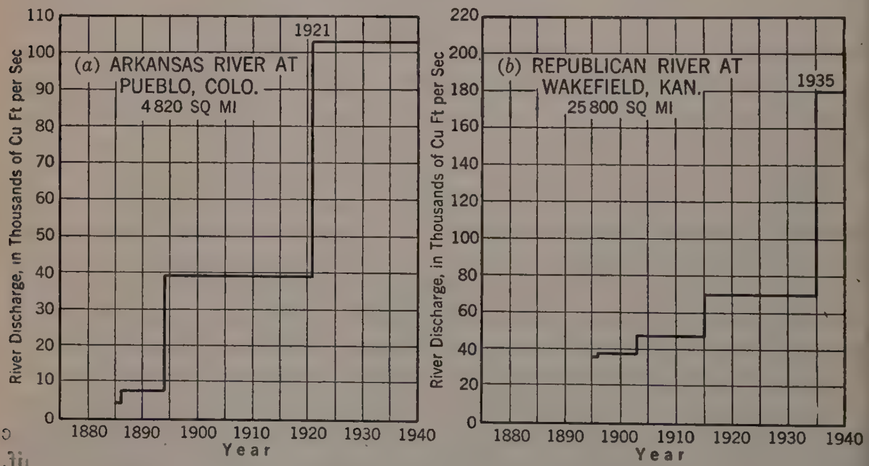

FIG. 2.—MAXIMUM FLOOD OF RECORD, TO AND INCLUDING ANY YEAR

On page 110, lines 1 and 8, substitute "discharge" for "stage."

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DETERMINATION OF KUTTER'S *n* FOR SEWERS PARTLY FILLED

Discussion

BY MESSRS. THOMAS R. CAMP, AND RICHARD G. COULTER

THOMAS R. CAMP,[11] M. AM. SOC. C. E.[11a]—The measured values of velocity and friction factor submitted by Mr. Johnson, for partly filled sewers, constitute a valuable contribution in a field where reliable measurements are very scarce. The data for Sewer A and Sewer D are of general interest because they apply to a circular cross section; and they are especially valuable because the depths are so small.

It has been known for some time (and the author has so stated) that the value of Kutter's *n* varies with the depth of flow in a sewer. Information on the manner of this variation has been so scarce, however, that it has been necessary in design to assume a constant value of *n* regardless of the depth of flow. This practice is illustrated in the shapes of the curves for the "hydraulic elements" appearing in most textbooks on sewer design; in which the velocity for a half-full circular pipe is shown to be the same as the velocity for the full pipe (see the broken line curve in Fig. 9).

In measuring the friction factor in a sewer, the experimenter has no control over the discharge and must accept the flow at the time of the measurements. Therefore, the scarcity of measurements for any substantial variation in depth for the same sewer is quite understandable. In fact there was no substantial variation in depth during Mr. Johnson's measurements, and the conclusion that *n* varies with depth hardly can be drawn solely from his recorded observations.

It is to be regretted that the opinion is so widely held that measurements made in pipes carrying clean water are not applicable to the flow of sewage. Such is not the case for clean sewers. The amount of suspended matter normally present in sewage is so small that it has a negligible effect on the flow pattern in the pipe. The friction factor is a function only of the Reynolds number and the pipe roughness, and therefore measurements made with clean water are equally applicable to sewage flowing in the same pipe.

NOTE.—This paper by C. Frank Johnson, Assoc. M. Am. Soc. C. E., was published in February, 1943, *Proceedings.*

[11] Associate Prof., San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[11a] Received by the Secretary February 25, 1943.

Measurements such as those made by Mr. Johnson are of limited value unless the results can be used to predict what the friction factor will be at other depths of flow. His estimates for the values of Kutter's *n* for the sewers flowing full are not substantiated by any experimental measurements, and they throw no light upon the value of *n* for other depths.

The best experimental measurements for sewers of circular cross section flowing partly full appear to be those of E. R. Wilcox[12] on 8-in. clay and concrete sewer pipe and S. M. Woodward, Hon. M. Am. Soc. C. E., and the late D. L. Yarnell, M. Am. Soc. C. E.[13] on open-butt-joint concrete and clay drain tile 4 in. to 12 in. in size. In Mr. Wilcox's work the pipes were clean and were laid carefully with good joints. The drain tile used by Messrs. Yarnell and Woodward was placed carefully in earth but with open-butt joints. The depth ranged from about ¼ full to full, in these experiments. The results showed conclusively that the friction factor decreases as the depth of flow increases, but the data were so erratic that reliable and accurate trends of friction factor with depth were not apparent.

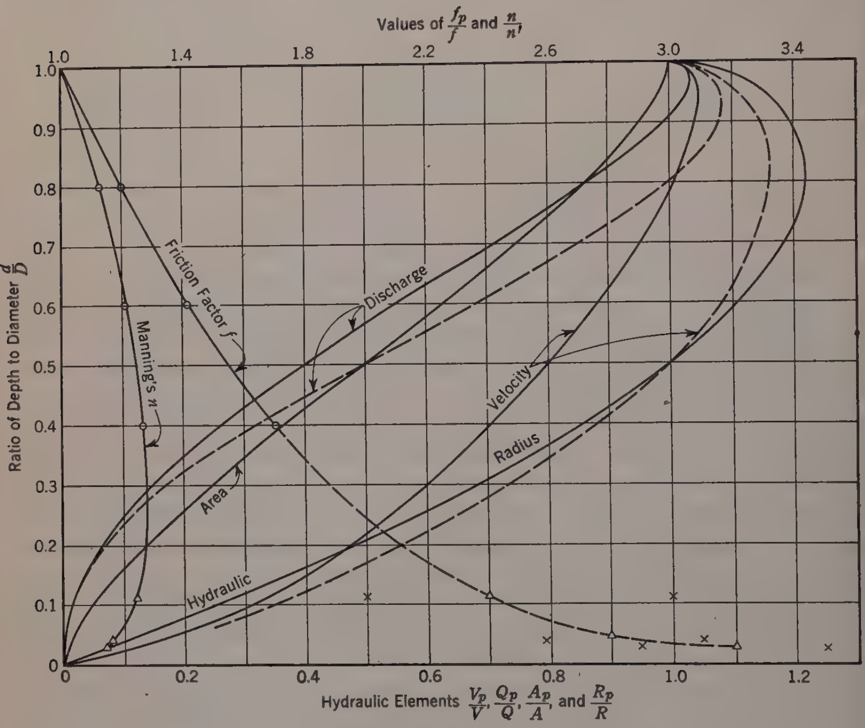

FIG. 9

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[12] "A Comparative Test of the Flow of Water in 8-Inch Concrete and Vitrified Clay Sewer Pipe," by E. R. Wilcox, *Bulletin No. 27*, Univ. of Washington Eng. Experiment Station, March 1, 1924.

[13] "The Flow of Water in Drain Tile," by D. L. Yarnell and S. M. Woodward, *Bulletin No. 854*, U. S. D. A., 1920.

The writer undertook a study of these data[12,13] in an effort to find a trend of friction factor with depth.[14] The average value of the Weisbach-Darcy friction factor f in Mr. Wilcox's work was about 0.019 (Manning's $n = 0.0095$) for 8-in. vitrified sewer pipe flowing full and about 0.022 (Manning's $n = 0.0102$) for 8-in. concrete sewer pipe flowing full. In the experiments of Messrs. Yarnell and Woodward, the average values of f and Manning's n for the pipes flowing full were as shown in Table 6. These values are much less than those generally used in practice. (The subscript p herein denotes "partly full.")

TABLE 6.—RATIOS OF FRICTION FACTORS $\frac{f_p}{f}$ FOR VARIOUS DEPTH-DIAMETER RATIOS

Pipe size (inches)	Friction factor, f	Manning's n	DEPTH RATIOS, d/D		
			0.8	0.6	0.4
4	0.033	0.0111	1.28	1.8	....
5	0.028	0.0106	1.15	1.34	1.9
6	0.027	0.0107	1.22	1.50	1.9
8	0.025	0.0108	1.18	1.38	1.7
10	0.023	0.0108	1.06	1.12	1.32
12	0.021	0.0107	1.21	1.40	1.70
Average	....	....	1.20	1.42	1.70

A study of the ratios of the values of the friction factor for partly full f_p and full pipes f in these experiments indicates a trend as shown by the points marked with circles in Fig. 9. These points represent the average results of 824 experiments by Messrs. Yarnell and Woodward on well laid drain tile, both concrete and clay. The average values of $\frac{f_p}{f}$ for each pipe size for slopes varying from 0.0005 to 0.015 are shown in Table 6.

The values of the ratios $\frac{f_p}{f}$ shown from the experiments of Messrs. Yarnell and Woodward as well as from those of Mr. Wilcox were quite scattered; but the averages for each ratio of depth to diameter, $\frac{d}{D}$, checked closely for both studies, the maximum deviation being about 5%. The experimental data seemed to show no consistent influence of the pipe material or of the variation in pipe size or slope. Mr. Wilcox used slopes of 0.005 to 0.04, so that the range of slopes for both sets of data was 80 to 1.

A study of the effective roughness projection k in these pipes was made from the von Kármán-Nikuradse[15] theory of turbulent flow in full pipes. The value of the friction factor f as given by Theodor von Kármán, M. Am. Soc. C. E., is, for smooth pipe—

$$\frac{1}{\sqrt{f}} = -0.8 + 2 \log (R \sqrt{f}) \dots\dots\dots\dots \tag{2a}$$

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[14] "Final Report of the Committee to Study Limiting Velocities of Flow in Sewers, March 4, 1942," *Journal*, Boston Soc. of Civ. Engrs., October, 1942, p. 286.

[15] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938.

and, for rough pipe—

$$\frac{1}{\sqrt{f}} = 1.74 + 2 \log \frac{D}{2\,k} \quad \text{(2b)}$$

in which $\mathbf{R}$ = the Reynolds number = $\frac{V\,D}{\nu}$ and k = the roughness projection on the pipe wall. The effective roughness projection on the walls of the pipe for the experiments of Messrs. Yarnell and Woodward is about 0.4 mm from Eq. 2*b* and is somewhat less for the pipe used by Mr. Wilcox.

A fairly good estimate can be made of the values of f for Mr. Johnson's Sewer D, and for Sewer A considered as a circular conduit, by guessing at the effective roughness projection k. From the description and from Fig. 2, a value of 0.5 mm seems reasonable for k, in Sewer A. From Eq. 2*b*, the corresponding value of f is 0.0117, assuming that turbulence is fully developed in the boundary film. For a smooth pipe, the value of f for Sewer A would be 0.0081 from Eq. 2*a*. An investigation of the thickness of the laminar film for the smooth pipe indicates that turbulence is not fully developed for k = 0.5 mm. Hence a somewhat smaller value of f, say 0.011 (Manning's n = 0.0125), is selected for Sewer A. From the photograph of Sewer D (Fig. 7), a value of $\frac{1}{4}$ in. seems reasonable for k; and, since turbulence is fully developed near the wall, the corresponding value of f is 0.038 (Manning's n = 0.016).

The writer has computed the values of f, from the author's measurements, and the corresponding ratios $\frac{f_p}{f}$ based on the foregoing values of f. For Sewer A, the average value of f_p for runs 1, 2, 4, and 5 is about 0.031 for $\frac{d}{D} = 0.042$, the corresponding value of $\frac{f_p}{f}$ being 2.8. For Sewer A, if $\frac{d}{D} = 0.028$, f_p is about 0.035, and $\frac{f_p}{f} = 3.2$. For Sewer D, if $\frac{d}{D} = 0.11$, the average value of f_p is about 0.09, and $\frac{f_p}{f} = 2.4$. The points corresponding to these values of $\frac{f_p}{f}$ are shown as triangles in Fig. 9, and the previously constructed curve has been projected through them.

For Sewer D, if the effective roughness projection k is $\frac{1}{2}$ in. or $\frac{1}{8}$ in., the corresponding values of $\frac{f_p}{f}$ are 2.0 and 3.0, respectively. For Sewer A, if f is taken at 0.010 or 0.012, the corresponding ratios $\frac{f_p}{f}$ are 3.1 and 2.6 for $\frac{d}{D} = 0.042$, and 3.5 and 2.9 for $\frac{d}{D} = 0.028$. These points are indicated as × in Fig. 9 for comparison with the selected values.

The effect of the variation of $\frac{f_p}{f}$ as shown by the curve upon the "hydraulic elements" is to displace the velocity and discharge curves from the positions indicated by the broken lines to the positions shown as solid lines.

The values of the ratio of Manning's coefficient,

$$\frac{n_p}{n} = \left(\frac{R_p}{R}\right)^{1/6} \sqrt{\frac{f_p}{f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

have been computed from the corresponding values of $\frac{f_p}{f}$ and are shown in Fig. 9. The positions of the curves $\frac{f_p}{f}$ and $\frac{n_p}{n}$ in this figure are based upon the best available experimental data, but these data are none too reliable for the purpose. Accurately controlled hydraulic laboratory work for a more precise evaluation of these functions is badly needed.

RICHARD G. COULTER,[16] JUN. AM. SOC. C. E.[16a]—In his work at Louisville, Ky., the author encountered a phenomenon reported by earlier investigators. In a given channel, Kutter's n tends to increase as the depth of flow decreases. In his conclusions, Mr. Johnson attributes the higher values of n at low flows to friction between the sewage surface and the air and the coating of slime and grease on the lower section of the sewer walls.

The results of a very comprehensive series of experiments on drain tile, 4 in. to 12 in. in diameter, with slopes varying from 0.0005 to 0.015 and with depths of flow varying from about one fourth of the diameter to full,[17] were reported by S. M. Woodward, Hon. M. Am. Soc. C. E., and the late D. L. Yarnell, M. Am. Soc. C. E., in 1935. These were laboratory experiments conducted with great care; but the results with flow at part depth varied considerably. They showed, in general, that the value of n was greater when the drain was partly filled than when entirely filled, the excess at one fourth of the depth being roughly from 10% to 50%; but there were a few cases in which lower values of n for less depths of flow were found.

Theodore Horton, M. Am. Soc. C. E., published the result of gagings in the sewers of the North Metropolitan Sewerage System of Massachusetts.[18] In this work n both increases and decreases, with increases in the depth of flow (hydraulic radius). In a description of the sewers gaged it is stated that: Deposits of grease and organic matter appeared on both sides of the channels, greatest near the line of average flow; and there was practically no deposit on the bottom of the channels (probably due to the cleansing action of the sand being swept along).

Mr. Horton's results and description show that the channel condition influences n in operating sewers. The investigations of Messrs. Woodward and Yarnell, however, were on clean pipe in the laboratory. In their work, air friction may have some, but probably only slight, influence on the increasing value of n.

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[16] Draftsman, C. C. Combs Organization, New York, N. Y.

[16a] Received by the Secretary March 9, 1943.

[17] "The Flow of Water in Drain Tile," by D. L. Yarnell and S. M. Woodward, *Bulletin No. 854*, U. S. D. A., 1935.

[18] "Flow in the Sewers of the North Metropolitan Sewerage System of Massachusetts," by Theodore Horton, *Transactions*, Am. Soc. C. E., Vol. XLVI (December), 1901, p. 78.

A more basic explanation of the variation in n with change in depth of flow may be obtained by recalling the laws of hydraulics used to derive the Chézy formula, Eq. 1. Fundamentally the resistance to flow is directly proportional to the frictional surface and variable with the velocity of flow in the relation, frictional resistance $= f\, V^y$. In addition, internal resistance in the fluid varying with the viscosity and diameter of the conduit must be taken into account. The effect of the diameter being inversely proportional to power, m, of the diameter.

Thus frictional resistance F, taking into account all the experimentally determined facts, becomes:

$$F = K \frac{V^y}{d^m} \qquad (4)$$

From this the elemental formula for difference in elevation of the hydraulic gradient (or surface of stream) is derived:

$$h = K \frac{l\, V^y}{d^x} \qquad (5)$$

in which K = a constant for wall friction; y = the coefficient of velocity relative to wall friction; and x = the coefficient of diameter or hydraulic radius relative to internal friction.

To change the form of Eq. 5: $V = \left(\frac{1}{K}\right)^{1/y} d^{x/y} \left(\frac{h}{l}\right)^{1/x}$; $V = K'\, d^\alpha \left(\frac{h}{l}\right)^\beta$;

or

$$V = C\, R^\alpha\, S^\beta \qquad (6)$$

In Eq. 6 the coefficient C is constant for a channel of given material when α and β are assigned correct values. (Formulas giving varying exponents for different classes of channels have been introduced.[19]) By inspection, it will be seen that α and β are influenced by the velocity, friction, and diameter or hydraulic radius.

In the Chézy formula α and β are assumed constant at 0.5. Therefore, C is no longer a constant indicating channel friction; it becomes a variable chiefly influenced by roughness, modified by slope and cross section to compensate for errors in the exponents assigned to R and S.

The empirical Kutter formula, devised to obtain values for C, contains the variables n, R, and S, with n as a constant for roughness and R and S to correct for the conditions of flow. However, R and S have constant exponents when their inclusion is necessitated to correct for exponential variation. The result, when C is determined for various depths of flow in a given channel solving the Kutter formula for n, causes n not only to indicate roughness but also to reflect the changes in hydraulic radius.

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[19] "Handbook of Hydraulics," by Horace W. King, 3d Ed., 1939, pp. 270–271.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PERMEABILITY OF MUD MOUNTAIN DAM CORE MATERIAL

Discussion

By Messrs. J. E. Christiansen and Milton Fireman, and William P. Creager

J. E. Christiansen,[9] Assoc. M. Am. Soc. C. E., and Milton Fireman,[10] Esq.[10a]—Differences in permeability that can be obtained merely by varying the soil-moisture content at which the material is compacted are emphasized in this interesting paper. They reveal a phenomenon that probably has not received sufficient attention and about which very little has been reported in the literature on this subject. The tests reported show variations in permeability as high as 10,000 to 1, caused by differences of only 1% or 2% in soil-moisture content at the time of compaction in the permeameter.

It is the purpose of the writers to call attention to another factor, generally overlooked, that also can cause very large differences in permeability values of soils—namely, the chemical composition of the water used for the permeability test. The writers have been primarily interested in the permeabilities of agricultural soils as related to infiltration of rain and irrigation water and to soil drainage. In these studies, the soil in the permeameter has not been compacted highly, but has been brought approximately to the field density of the soil. It is possible that the large differences in the permeabilities of these soils, resulting from differences in quality of water, would be reduced greatly in magnitude if the materials were compacted more highly, but, as yet, no tests have been made to support this possibility.

In preparing the samples for testing, the soil is first air-dried and sieved, poured into the permeameter, leveled, and then jarred a specified number of times to bring the materials to a fairly uniform density. Next, the soils are slowly wetted from below until the soil mass is saturated and most of the air

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Note.—This paper by Allen S. Cary, Assoc. M. Am. Soc. C. E., and Boyd H. Walter, and Howard T. Harstad, Juniors, Am. Soc. C. E., was published in September, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1942, by F. H. Kellogg, M. Am. Soc. C. E.

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[10] Junior Soil Scientist, U. S. Regional Salinity Laboratory, Bureau of Plant Industry, Soils, and Agricultural Eng., Agricultural Research Administration, U. S. Dept. of Agriculture, Riverside, Calif.

[10a] Received by the Secretary March 1, 1943.

has been expelled. Finally, the water is admitted to the top of the soil column; it passes downward through the soil and is caught and measured in a graduated cylinder.

Distilled water was used in the studies reported by the authors. Some of the soils tested at the Regional Salinity Laboratory in Riverside, Calif., are several hundred times more permeable to local tap water (which contains about 280 ppm of mixed salts—that is, approximately 34 ppm calcium, 35 ppm sodium, 6 ppm magnesium, 16 ppm chloride, 36 ppm sulfate, and 153 ppm bicarbonate) than they are to distilled water, or to waters containing a high percentage of sodium salts. As is well known, the colloidal complex of most agricultural soils of the West is mainly calcium saturated; that is, calcium is the principal absorbed cation. In general, these soils are reasonably permeable to water and have a suitable physical condition for cultivation and crop production. When water containing a high percentage of sodium salts is passed through these soils, base exchange takes place. The sodium ions replace a large part of the absorbed calcium ions, and the soils may become deflocculated, sticky, jellylike, and much less permeable to water. When leached with high sodium water, some soils remain moderately permeable as long as the salt content is fairly high, because the presence of salt tends to keep the soil floc-

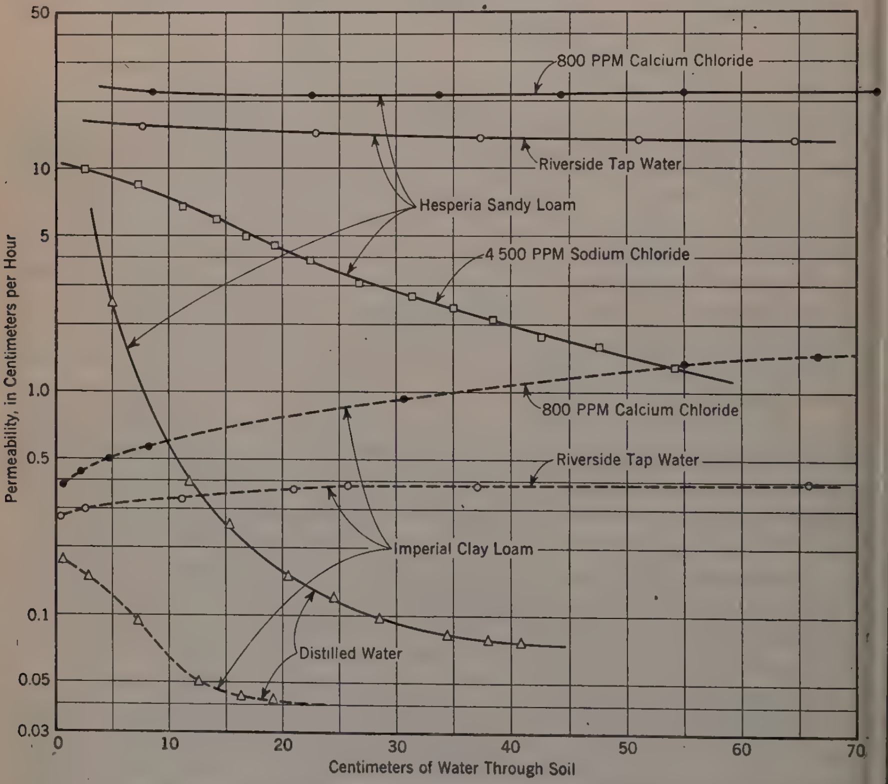

FIG. 10.—PERMEABILITIES OF TWO AGRICULTURAL SOILS AS INFLUENCED BY WATER COMPOSITION

culated. If the salt solution is replaced by a water of considerably lower salt concentration, the soil will disperse immediately and become relatively impermeable.

Characteristic permeability curves for two agricultural soils are shown in Fig. 10. The curves show the changes in permeability that occurred with the amount of water which passed through the soil. The Hesperia sandy loam is a surface soil from near Bakersfield, Calif., and the Imperial clay loam is from near El Centro, Calif. These soils are typical of many others tested in that they show marked differences in permeability to distilled water as compared with ordinary tap water, or with waters containing only a few hundred parts per million of calcium salts. It will be noted that the Hesperia soil is more permeable to a solution of sodium chloride than it is to distilled water. However, had this soil been subsequently leached with distilled water, it would have been practically impermeable. One soil tested was 35,000 times as permeable to a water containing 800 ppm of calcium chloride as to distilled water.

Therefore, there may be a danger in using distilled water for testing the permeability of soils used for structural purposes. In place in a dam or other embankment, soils are subjected to the percolation of natural soil waters containing dissolved salts. They may be many times more permeable to these soil solutions than to distilled water.

William P. Creager,[11] M. Am. Soc. C. E.[11a]—For impervious materials an amount of moisture should be provided which is slightly greater, rather than slightly less, than optimum, particularly when natural moisture content is deficient and more must be added. This fact is indicated strikingly in this paper.

It is understood that the laboratory tests of the Mud Mountain Dam core material showed no tendency to swell when saturated subsequent to loading. Although the montmorillonitic fraction of the soil is subject to swelling, it constituted only about 1% of the total volume.

However, the following description of the expected behavior of a soil subject to swelling is so similar to that observed at Mud Mountain that it is felt that, although the Mud Mountain soil did not swell, there must have been some as yet undiscovered action allied to that produced by swelling. The following discussion applies to a soil subject to swelling.

It is interesting to trace the behavior of a soil swelling having different amounts of moisture when applied on the dam or in the laboratory: For a small amount of applied moisture, the applied moisture is not well distributed throughout the soil, the material is lumpy, and no amount of churning or compaction can distribute the moisture uniformly. Thus, a fraction of the material has too little; and another fraction too great, an amount of moisture. The fraction that has too little moisture may be subject to swelling when subsequently saturated, with resulting increase in voids and loss of strength.

For instance, consider Fig. 3 with 16.5% of applied moisture. If one fraction of the material contains only 13.7% moisture and the other fraction

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[11] Cons. Engr., Buffalo, N. Y.

[11a] Received by the Secretary March 15, 1943.

19.6% moisture and the total dry weight is 120 lb, the fraction containing only 13.7% moisture is subject to swelling, increased voids, greater permeability, and loss of strength.

However, if 19.6% of moisture were applied to the total mass, the dry weight still would be 120 lb, but the material would not be subject to swelling with consequent loss of desirable attributes. As the amount of moisture is increased, the distribution will be much more uniform; swelling and permeability have decreased and strength has increased.

When the moisture is increased to slightly greater than optimum, swelling has been practically eliminated, and the material has reached its optimum of density and strength after subsequent saturation, as shown by Fig. 5(*a*) for a 20% till admixture which has an optimum moisture of 16.5%. The use of an abundance of moisture, however, is obviously deleterious, as shown by the reversal of the permeability curves of Figs. 5.

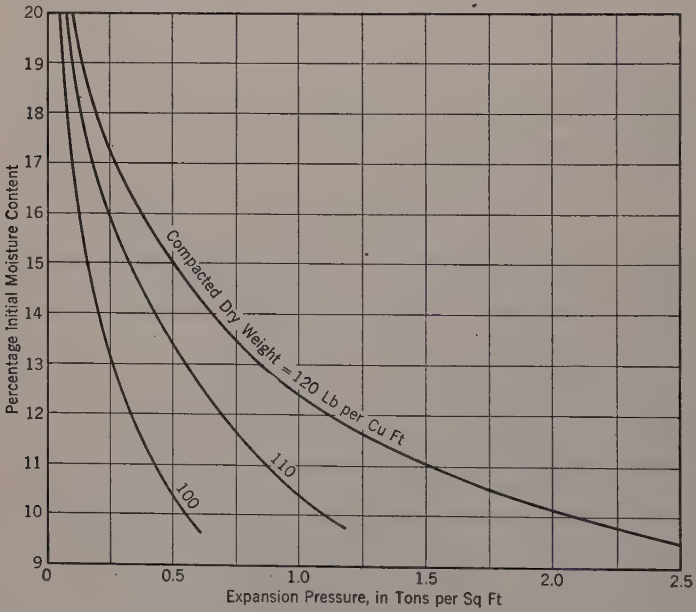

FIG. 11

The foregoing discussion applies, of course, to those many materials which, when placed with deficient moisture, are subject to swelling when subsequently saturated.

Typical characteristics of an expansive soil are shown in Fig. 11. Even for a constant density, the pressures required to prevent expansion decrease as the initial moisture content increases.

It is also pertinent to note that, for any given material and moisture content, such expansion would not occur if there were sufficient resistance to expansion offered by the material placed in the dam adjacent to and above it. In such cases, the permeability would be less, and strength would be greater, than indicated by the tests.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

CONCRETE RESERVOIRS OF THE VERTICAL-BEAM TYPE

Discussion

By Messrs. Victor H. Cochrane, and R. E. Koon and J. C. Gearhart

Victor H. Cochrane,[3] M. Am. Soc. C. E.[3a]—The clear presentation of the advantages and limitations of the vertical-beam type of reservoir and the discussion (based on the author's experience) of both desirable and faulty details of construction make this paper a valuable contribution.

A reservoir of this type, built in the spring of 1942 for a large army cantonment in the Southwest, was designed under the direction of the writer. The capacity of 3,000,000 gal is sufficiently large to permit the elimination of conventional steel elevated-tank or standpipe storage. As the reservoir is located on a hill of suitable elevation, the shape of the limited area on top dictated the use of a circular reservoir. Earth was used for backing and for a roof cover 18 in. in thickness in order to improve the quality of the stored water, to minimize temperature variations, and to effect concealment of the structure. The inside diameter is 164 ft and the depth of water, 20 ft. The rock face of the foundation trench is relied upon to resist that part of the bottom reaction of the wall sections not taken care of by the earth backing. The support of the top of the wall is similar to that shown in the middle detail of Fig. 3(*a*). There are twelve vertical joints at intervals of 43 ft. The weight of reinforcement required was found to be only about one half as great as for a conventional hoop-tension reservoir.

This structure departs from the author's recommendations in the following particulars: (1) It is circular in plan, for the reason previously stated; (2) it is covered and backed with earth; and (3) the length of the wall sections is greater than recommended. In general, these departures may be explained by the

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Note.—This paper by C. Maxwell Stanley, Jr., M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Lewis A. Schmidt, Jr., M. Am. Soc. C. E.

[3] Cons. Engr., Tulsa, Okla.

[3a] Received by the Secretary February 19, 1943.

fact that the reservoir was expected to be in use not more than five years, thus making the requirements less severe than for a permanent structure.

From the structural point of view the earth covering is advantageous in some respects as it greatly reduces the temperature range and, at the same time, protects the concrete from the effects of freezing and thawing. Since the inward earth pressure is about one half as great as the outward water pressure with full reservoir, the inner face of the wall will remain in compression except when the tank is empty or drawn down to an unusual extent. If some small cracks should appear under these conditions, they should close when the reservoir is filled above the normal drawdown level.

Experience has shown that, unless extraordinary measures are employed, a concrete wall resting on earth or rock and exposed to extreme ranges of temperature tends to crack at intervals of 15 ft to 20 ft, and therefore the statement that the wall sections should be less than 20 ft long between joints is correct. As the author suggests, horizontal bending stresses may be set up in long sections because the vertical-beam elements are somewhat restrained from bending radially. The section is strengthened by this action unless cracking occurs. In the present instance it was felt that the earth protection and the short period of use justified the joint spacing adopted.

The structure has been in use since June, 1942, and no defects of consequence have been reported. The circular structure is free from the undesirable conditions occurring at the corners of rectangular reservoirs. It will be interesting to note whether any objectionable cracking occurs.

In his discussion of retaining wall types (see heading, "Conventional Designs: Retaining Wall Types"), the author omits reference to the arch-wall design, such as that developed by the writer for a 1-million gal clear well of the water treatment plant near Fort Smith, Ark. Several descriptions of this reservoir have been published.[4] This type of structure seems to eliminate all of the objectionable features listed by the author, and, in addition, it has some advantages.

The Fort Smith reservoir is circular, with a 20-ft depth of water, and the wall consists of twelve arches of 25-ft span, 8 in. in thickness, springing from radial buttresses with counterfort extensions. Foundations are in hardpan, and the structure is earth covered. The arches preferably are poured in stages, the forms being raised as the concreting progresses. The entire wall, including the joints between arches and buttresses, is in compression. Experience with multiple-arch dams indicates that such joints are not likely to leak, and, since there is no tendency for the joint to open, simple and inexpensive stopwater elements may be used. The only joint requiring special treatment is that between arches and footings. Temperature effects are taken care of by the flexibility of the arches. The roof is of beam and slab construction, with exterior columns located at the heel of the counterforts to lend stability to the wall.

A design was also made for a pre-stressed tank with a domed roof. The quantities required for the two designs were found to compare as follows:

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[4] *Engineering News-Record*, September 17, 1936, p. 400; also *Water Works Engineering*, August 19, 1936, p. 1068.

	Arch-wall design	Pre-stressed wall design
Reinforcement, lb.	94,000	112,000 (including turnbuckles)
Concrete, cu yd.	672	650

For an aboveground reservoir of the arch-wall type the quantities required would be considerably less than the foregoing. The Fort Smith tank has been in service since 1936, and no leakage has been observed and no defects have developed.

This kind of reservoir is unlimited as to size and virtually unrestricted as to shape. The roof may be of any construction, or it may be omitted entirely. The greater the capacity, the less will be the cost per unit of storage. If a cover is provided, a square or rectangular shape is preferable.

The economic depth for square arch-wall reservoirs is given by the formula

$$h = \left(\frac{C_1}{k}\right)^{2/5} \frac{V^{1/5}}{2} \quad \text{. . . . . (1)}$$

in which h = depth of water in feet; C_1 = cost of floor, roof, and roof covering, in dollars per square foot of reservoir area; V = volume of storage in cubic feet; and k is a constant.

The cost of the wall varies approximately as the square of the height, and $k = \frac{C_2}{h^2}$ (assumed), C_2 being the cost of the arches, buttresses, and backfill (if any) per linear foot of wall. Table 2 shows typical variations in economic depth for covered and uncovered reservoirs for assumed values of C_1 and k. The depth may be varied either way from the most economic value without greatly increasing the cost.

TABLE 2.—ECONOMIC DEPTHS FOR SQUARE ARCH-WALL RESERVOIRS

Reservoir	Cost C_1 (dollars per sq ft)	Constant k	CAPACITY, IN MILLION GALLONS:							
			0.1		1.0		3.0		10.0	
			Depth h	Length[a] l	Depth h	Length[a] l	Depth h	Length[a] l	Depth h	Length[a] l
Volume V (cu ft) . . .	. . . .	. . . .	13,400		133,700		401,000		1.337.000	
Covered	1.50	1/10	9.9	37	15.7	93	19.5	144	24.8	232
Open	0.50	1/12	. . . .	. . . .	10.9	111	13.6	172	17.2	279

[a] Length of side; all dimensions are in feet.

R. E. KOON,[5] M. AM. SOC. C. E., AND J. C. GEARHART,[6] ASSOC. M. AM. SOC. C. E.[6a]—Having, presumably, investigated the merits and deficiencies of covered concrete reservoirs built according to "conventional design," the author concludes that such structures can be built, to advantage, with "vertical-beam" side-walls. This conclusion was reached as the result of theoretical analyses and actual construction experience. Only the vertical-beam type is detailed in the paper, but it may be inferred from the text that the "conventional" type had been tried with unfavorable results. The author classes as "conventional" cantilever walls or buttress-supported walls and also hoop-reinforced walls of round reservoirs except those wherein the steel is prestressed. This definition seems adequate and is accepted for purposes of reference herein.

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[5] Cons. Engr. and San. Engr. (Stevens & Koon), Portland, Ore.

[6] Ensign, CEC-V(S), U.S.N.R., Camp Peary, Williamsburg, Va.; formerly Asst. Engr., Stevens & Koon, Portland, Ore.

[6a] Received by the Secretary March 23, 1943.

This discussion deals with certain of the author's conclusions which the writers cannot accept as well founded in theory, or established as the result of their own practical experience.

General Comments.—The data presented in support of the author's conclusions are limited, in all practical aspects, to observations on one reservoir of moderate size (3 million gal) in which several defects appeared, and to two quite small reservoirs (0.12 million gal and 0.10 million gal, respectively) in which changes in details of design were made to correct for defects observed in the earlier structure.

In general, the writers have favored the author's so-called "conventional design." An exception has been made in the case of relatively small structures, mainly settling basins ahead of mechanical filters, where the lengths of tie beams have been rather short. As a consequence of these experiences, this paper favoring vertical-beam construction for reservoir walls has been read with much interest.

Adverse Conclusions.—The writers' conclusions are quite contrary to those reached by Mr. Stanley. The importance of his assumptions of deterioration of concrete-embedded steel also is questioned. Supporting and explanatory discussion will follow, but an outline of these adverse conclusions is summarized thus:

1. Except under certain unusual conditions, there will be no advantage in the "vertical-beam" walls either structurally or economically;

2. To assume that defects which appeared in a 3 million gal reservoir have proved to be corrected fully by changes made in design of later structures, each of which is less than one twenty-fifth the size of the original structure, is not convincing;

3. Structures of "conventional" design are not more subject to leakage than are those of the vertical-beam type;

4. It is not necessary to apply surface waterproofing to walls of "conventional" type which have been built properly (this is also true of walls of the vertical-beam type);

5. No evidence has been produced by the author, nor have the writers found it in literature, to the effect that water entering the minute cracks (which theoretically appear in properly stressed concrete walls) will cause injury to reinforcing steel;

6. The use of vertical-beam walls in place of cantilever walls does not effect a simplification of design; nor does it afford appreciable advantage to the builder;

7. Steel tie rods required by vertical-beam construction, unless specially protected, are more vulnerable to deteriorating action than concrete-encased reinforcement as used in walls of "conventional" design;

8. The writers find that an analysis of relative costs favors "conventional" design in the ratio of about 5 : 4;

9. Application of the vertical-beam theory to round reservoirs would develop extraordinary problems in tie-rod arrangement which should preclude any attempt to use such a design; and

10. Although the design of a reservoir with vertical-beam walls may be theoretically sound, the possibilities of major structural failures due to defective workmanship, particularly with respect to tie-rod placing and adjustment, seem very great as compared with an equally well-designed structure with walls of the "conventional" type.

Basic Theory.—The author's theory, that the use of an adequately supported vertical beam to carry the load due to water pressure in a reservoir is feasible, cannot be disputed; but the application of this theory cannot be made properly to reservoirs of all ordinary sizes and shapes. In some instances, a reservoir must be built (*a*) of small capacity, (*b*) with necessarily high vertical walls, (*c*) square or rectangular in plan, and (*d*) where the steel tie members can be encased in good concrete. In these cases, the writers agree with the author as to the soundness of application of the theory but only if its use will show economy in construction cost as compared to a design of "conventional" type. In short, a good reservoir can be built using either type of wall design. Architectural requirements or foundation conditions may be determining factors in the selection of the type of wall to build. In another case the designer's preference, influenced by his personal appraisal of the relative merits of compression or tension in the wetted wall face, may determine the final choice.

Few reservoirs, however, must conform in all respects to the foregoing stipulations. Usually the designer has several options: The reservoir may be relatively shallow and consequently may cover a greater area; it may be made round in plan and relatively shallow or deep; and it may be built with a considerable part, or all, of the water capacity below or above natural ground level. All, or some, of these alternates may be, and usually are, available for the designer's choice, and his determination to give predominant weight to any special condition perhaps will be influenced by many factors. In some instances, certain unalterable facts—the elevation of the available site in relation to a fixed requirement for elevation of the water, the nature of the foundation or of the material to be excavated, architectural requirements, and perhaps others—may be conclusive in dictating the type of reservoir wall to be built; but such circumstances are rare. For purposes of this discussion, it is understood that certain basic requirements only remain unchanged: (*a*) The reservoir must be built of reinforced concrete; (*b*) it is to be provided with a roof—presumably of concrete, but not necessarily so; (*c*) it must have the required capacity; and (*d*) it must be a well-built structure, free from leakage.

Only rarely will there be such a combination of all circumstances as clearly will favor the designer's selection of the vertical-beam type of wall; and, consequently, the application of this "basic theory" of vertical-beam construction to all concrete-covered reservoirs or, indeed, to more than a negligible few, is not sound.

Effect of Cracks in Concrete.—It is regrettable that so little information has been published on the appearance of cracks, either visible or microscopic, in reinforced concrete members under normal loads. The stresses within a vertical wall subjected to hydraulic loading can be calculated very precisely, and, consequently, the deformation of such a wall can be determined with reasonable accuracy. Since the tensile strength of concrete alone is assumed as zero in most reinforced concrete design, it follows that minute cracks must occur in any member that suffers deformation; and deformation will occur, of course, as soon as any load is applied.

The author's principal argument in favor of the vertical-beam type of wall for reservoirs seems to be that the entire concrete surface in contact with the water is thus in compression and therefore free of cracks which might permit moisture to attack the reinforcing steel. Although the writers agree that minute cracks will develop in the inside face of a cantilever wall, they are not convinced that these cracks are of sufficient magnitude to permit water to enter and thus, perhaps, to be detrimental to the steel, unless such a wall is loaded appreciably above that allowable in normal design.

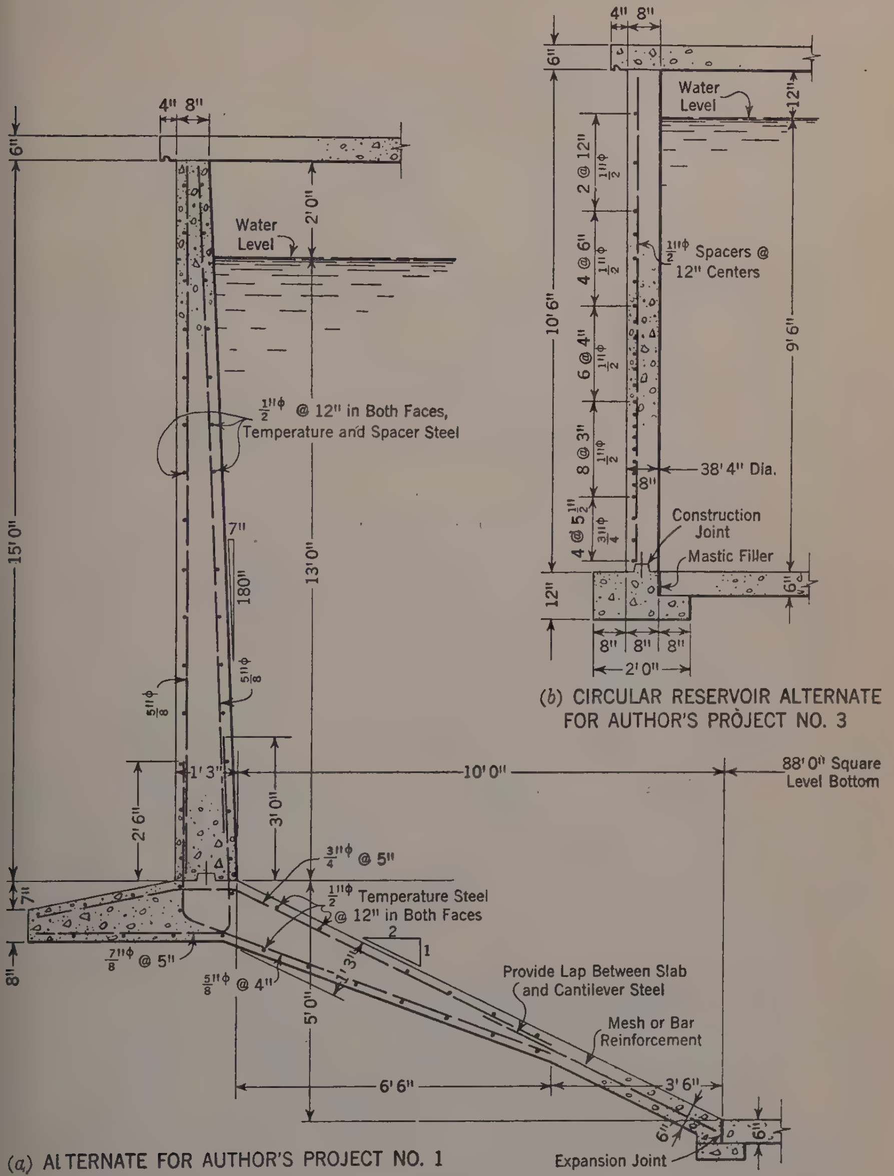
If it is assumed, because there is no evidence to the contrary, that the cracks in a cantilever wall will be large enough to permit water to contact the steel, it is evident that similar cracks of equivalent magnitude must appear on the outer or tension side of the wall designed as a vertical beam. Such cracks, although not exposed to water from within the reservoir, would be vulnerable to the attacks of rain, air, and frost if the wall is exposed to the atmosphere and perhaps even to worse conditions if the wall supports a backfill. These agents may be much more severe in their effect on the reinforcement and concrete than the reservoir water alone.

By the use of sufficient reinforcing steel, all cracks can be eliminated or so reduced in size as to remove any chance of their being deep enough to reach the steel. There is no evidence, however, to indicate that such extravagance in steel could be justified, in any way, whether the wall be of the "conventional" or the vertical-beam type.

Choice of Design.—The use of walls of the vertical-beam type in large reservoir construction may be new, but this does not signify that they have not been considered for such service. The experienced reservoir designer customarily has in mind several alternate types of structures or methods of construction for each such project; and, since there are frequent occasions to use vertical beams to good advantage in ordinary structures, it is not reasonable to assume that their possibilities for reservoirs would be overlooked. It appears, therefore, that the chief reason why this type of design is not usual, and consequently "conventional," is because safer or more economical types have been chosen.

Relative Cost of Construction.—It is stated in the paper that reservoirs with vertical-beam type of walls have been constructed without increase in cost over other types. A few calculations made by the writers do not appear to bear out this statement. Alternate designs for the author's project No. 1 and project No. 3 are submitted herewith in Fig. 6.

Since the author does not indicate the roof or floor slab reinforcement, nor the column supports for his roof in project No. 1, it is assumed that the quantities of steel would be substantially the same with either type of wall and, therefore, these quantities have been neglected in the comparisons made. In making the comparative estimates, it also has been necessary to assume dimensions which appear to be reasonable for structural details not given by the



(a) ALTERNATE FOR AUTHOR'S PROJECT NO. 1

(b) CIRCULAR RESERVOIR ALTERNATE FOR AUTHOR'S PROJECT NO. 3

FIG. 6

author. Likewise, it will be necessary to make similar assumptions for details which the writers could not show in their sketches.

The roof slab in Fig. 6 is shown, with a thickness of 6 in., to be comparable to the author's design for estimating purposes. This type of wall design requires slightly greater roof area. The vertical steel bars in the outside face, carried to varying elevations, are adequate for earth backfill as high as the water level, and should be omitted if there is no backfill or if they are not required as spacer bars to support temperature steel. The principal reinforcement bars in the wall and in the footing vary in lengths, as the stresses require.

The writers' alternate for project No. 1 (Fig. 6(*a*)) has a capacity of 1.5 million gal in each of two square sections, with a common wall between. This division wall is similar to the side-wall detail except that the floor has a 1-on-2 slope for a distance of 10 ft on each side.

The concrete quantity in project No. 1 (Fig. 5(*a*)), including walls, footings, and floor and roof slabs, approximates 1,895 cu yd. The reinforcing steel, neglecting that in the roof and floor slabs and assuming that no extra steel is added in the roof slab to restrain the top of the wall, is calculated to be 183,000 lb. If the concrete is estimated to cost $40 per cubic yard and the steel $0.06 per pound, then the total cost of these parts of the reservoir is approximately $87,000.

For the alternate design indicated in Fig. 6(*a*), a reservoir of equivalent capacity, with the same items of steel neglected, the concrete is estimated at 1,480 cu yd and the reinforcing steel at 131,000 lb. Using the same unit prices, the cost of these similar parts of the alternate reservoir is approximately $67,000, thus showing a saving of $20,000 in these items.

This comparison does not present quite a true picture, however. If the two reservoirs were located on identical sites, the alternate reservoir must be 5 ft lower than the all-vertical-wall type to obtain the same foundation medium. Approximately 4,000 cu yd of additional excavation would be required for the proposed alternate, but it still would show a saving of probably $16,000, or more than 18% in construction cost. Since local conditions affecting the available site elevation are as likely to favor one design as the other, this feature as related to probable cost cannot be considered here without undue extension of this discussion. However, by placing a series of foundation walls under the slope slab and wall footing and by increasing the thickness of the sloping slab to carry the load between these walls, the alternate reservoir could occupy any site suitable for the vertical-beam type; but there then would be little difference in cost of the two types. It should be noted here that the 131,000 lb of steel used in the alternate design include about 35,000 lb in the outer face of the cantilever wall to permit backfill to the full wall height. If such backfilling is not required, then an additional saving of $2,100 would be effected in favor of the "conventional" design.

In project No. 3 (Fig. 5(*c*)), the author again gives adequate details only of the wall, footing, and wall cap. Therefore, the quantities involved in these units are the only ones included in the following comparison. From the details given, the capacity of this reservoir is only a little more than 82,000 gal. There are 51.7 cu yd of concrete in the walls, footings, and cap, and the weight of the reinforcing steel (neglecting tie rods for which no data are given) is 9,200 lb. Using the same unit prices as before for comparative estimates, the cost of these parts of the reservoir would be $2,620.

For a circular reservoir of the same depth and capacity as shown for project No. 3 (Fig. 6(*b*)), the concrete, excluding roof and floor slabs, is 39.6 cu yd and the total weight of the reinforcement, with $f = 12{,}000$ lb, is 3,250 lb. The cost of wall and footing alone then would be $1,779 or 32% less than for these same features of project No. 3. The unit cost of concrete might be higher for the circular wall, but, even if this proved to be true, the round type has such distinct advantages for so small a structure that the writers certainly would select this type unless some very unusual local conditions should dictate another choice. In addition to the quantities compared, there is also the problem of tie rods and expansion joints in the vertical-beam-wall type, and their cost undoubtedly would be such as to permit the designer to double the quantity of reinforcement in the circular tank and stress the steel as low as 6,000 lb without showing an unfavorable cost comparison for the round type.

Vertical Construction Joints.—The author states that some difficulty with leaks through vertical shrinkage or contraction cracks between the construction joints occurred on his projects Nos. 1 and 2, and that, as a result, the subsequent design satisfactorily employed construction joints at 20-ft centers or less. However, since the final project listed had walls only 34 ft long, there is no practical reason to conclude that construction joints at 20-ft intervals are necessary or desirable. The writers are certain, as a result of their own experience, that such short wall sections would accomplish nothing in guarding against reservoir leakage, would increase costs, and perhaps actually would result in inferior work.

Many structural design authorities use round figures of 0.20% or 0.25% for temperature reinforcing in ordinary slab design. However, extensive tests and experiments by C. P. Vetter[7] and Walter Dreyer,[8] Members, Am. Soc. C. E., indicate that 0.60% to 0.75% of temperature steel is necessary to eliminate all possibility of cracks large enough to permit leakage from a concrete reservoir. Under conditions where maximum air temperature ranges are as much as 100° F, the writers have used materially smaller percentages of steel than here recommended, with entire success.

In project No. 1, the author used longitudinal temperature steel to the extent of only 0.17% of the cross-sectional area of the wall and, in project No. 2, it was approximately 0.14%. These low percentages of temperature reinforcing steel point quite definitely to a reason for the troubles experienced.

In the design of a 15 million gal reservoir (built in 1939) with a 20-ft water depth and wall section somewhat similar to that shown in Fig. 6, the writers used approximately 0.35% temperature steel in main wall sections, 58 ft 8 in. long between construction joints. After this reservoir had been in use for approximately three years, there were neither visible cracks nor leakage in any of these relatively long wall sections, and a face, 475 ft long, was exposed for its entire height. Experience on this and other reservoirs of somewhat similar design has led the writers to believe that the distance between construction joints can be almost any convenient length, within reason, as long as the amount of temperature steel is adjusted accordingly. A design feature of this reservoir, which may have contributed materially to the elimination of shrinkage cracks, was the separation of all major wall sections by short closing sections which were poured not less than 20 days after the adjacent wall sections were built.

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[7] "Stresses in Reinforced Concrete Due to Volume Changes," by C. P. Vetter, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1039.

[8] *Ibid.*, p. 1058.

The troubles experienced by the author with shrinkage or contraction cracks in his project No. 1 are definite evidence of the undesirability of making a rigid connection between wall and roof in a structure so large. From the incomplete details given, the reader must assume that the roof slab is one continuous piece, approximately 125 ft by 185 ft on its sides. A 100° range in temperature will produce material changes in slab dimensions and, consequently, transmit very severe stresses to the wall sections. In all cases the walls will be affected much less by changes in air temperatures than the roof slab and thus will not expand or contract in harmony with it. The best solution is to support the roof slab entirely free of the walls so that movements in one will not transmit stress into the other; at least this is true in structures of such size that vertical expansion joints are required in the walls.

Tie-Rod Use Questionable.—The use of unprotected tie rods seems ill-advised when they perform so important a function in the stability of the structure. Corrosion surely will be severe when bars are placed in earth or gravel under a reservoir floor or when they are suspended below the roof slab in what is always an extremely humid atmosphere. Such steel cannot be expected to approach the life of concrete-encased reinforcement. If so used, the safety factor should be increased materially, and this, in turn, increases the steel quantity. In some way, these tie rods may be encased or protected, but construction costs will increase with any such design provisions. The difficulties of properly placing and adjusting these tie rods, together with the uncertainties and hazards attendant on their use, seems so great as to preclude use of a design which requires them even if a substantial construction cost saving could be realized.

Surface Waterproofing.—The author states that the walls require a waterproofing treatment to fill the pores in the concrete, thus making it impervious to water. Test results have convinced the writers that this is not necessary and adds considerably to the over-all cost of the reservoir.

The 15 million gal reservoir previously mentioned is approximately 235 ft by 475 ft in plan, with a dividing wall to provide two square units. Although there is a 20-ft water depth when full, the floor slab is only 6 in. thick and the slope and wall sections are quite similar to those shown in Fig. 6(*a*). All joints between adjacent floor-slab panels, as well as wall sections, have flexible metal water stops embedded in the concrete. Definite artificial cracks, about ½ in. wide, were provided at the water stops in the floor, and these were filled with hot mastic after completion of the roof. (It is important that the roof be completed before the mastic joint-filler is poured as, otherwise, during and for some time after the joints are completed, the floor slabs are subject to expansion and contraction greatly out of proportion to that which occurs after the roof is in place. Unless this procedure is followed an inferior job of joint construction is quite certain to occur.) No waterproofing of any kind was applied to the concrete mix or its finished surfaces. The reservoir is underdrained completely to an open pit. At no time since the reservoir has been in service has there been any flow whatever from these drains. The soil under the reservoir is practically impervious, and, consequently, any leakage would be evidenced by flow in the drains.

Upon its completion, the reservoir was filled to capacity and subjected to a prescribed leakage test: As measured by hook gage, the water level dropped $\frac{1}{8}$ in. in 48 hr, thus indicating a loss of approximately 4,250 gal per day, a truly negligible part of the capacity. This test was repeated after the reservoir had been in service three years, and there was no measurable change in the rate of loss. Since there have been three cycles of seasonal changes in temperature, it seems reasonable to assume that this minute loss is established permanently and is certainly undetectable through visual observation of exposed walls or open underdrains. This case is cited thus because it proves on a rather large scale that even relatively great areas of thin concrete slabs are watertight if suitable concrete is used and, particularly, if adequate care is taken to insure its proper deposition and cure.

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DISCUSSIONS

CHARACTERISTICS OF HEAVY RAINFALL IN NEW MEXICO AND ARIZONA

Discussion

By Lawrence Pratt, Assoc. M. Am. Soc. C. E.

Lawrence Pratt,[9] Assoc. M. Am. Soc. C. E.[9a]—Engineers of the Southwest will find this paper a valuable compilation of 1-day rainfall records, supplementing the previous work of Mr. Yarnell.[2]

It has been observed at Tucson, Ariz., that, for a period of six to eight weeks during July and August, thunderstorms are of almost daily occurrence. A notable feature of these storms is the punctuality of their appearance. About 10:00 a.m. high cumulus "thunderheads" appear in the northwest and mount rapidly to the zenith. The first drops of rain may be expected about 2:00 p.m. but within another 2 hr the storm will have passed and the sun will be shining again. The first raindrops are very large, making splashes on the ground the size of silver dollars, but quickly becoming smaller. Probably the heaviest intensity during the storm occurs within the first 15 min. Each storm is accompanied by a sharp temperature drop of 20° to 30°, and there may or may not be a strong northwest wind.

Another type of storm which may be peculiar to southern Arizona and southern New Mexico is the "duster" which occurs in early and late summer. This is characterized by a persistent wind which at times reaches a high velocity and which carries great quantities of dust. The usual duration of this storm is 3 days. The wind increases in velocity from day to day and the storm culminates in a thundershower which may last no longer than 30 min. It is hoped that the author will explain the meteorological aspects of this type of storm.

From the summit of Bill Williams Mountain, 4 miles south, and 2,500 ft above Williams, Ariz., the writer had an opportunity to observe weather phenomena throughout one summer. On certain days as many as four

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Note.—This paper by Luna B. Leopold, Jun. Am. Soc. C. E., was published in February, 1943, *Proceedings*.

[9] Los Angeles, Calif.

[9a] Received by the Secretary March 1, 1943.

[2] "Rainfall Intensity—Frequency Data," by David L. Yarnell, *Miscellaneous Publication No. 204*, U.S.D.A., August, 1935.

separate and apparently independent thunderstorms were seen at the same time, moving from west to east over the plateau which reaches north to the Grand Canyon. These storms were of small extent, with sharply defined boundaries, covering from 1 to perhaps 4 or 5 sq miles.

Because of the limited area and short duration of high-intensity summer storms, the problem of deducing runoff from rainfall is complicated. Only the smaller drainage basins have times of concentration sufficiently short to coincide with the times of precipitation of the storms, although local flash floods may be produced occasionally even in streams of considerable drainage area. However, notable floods on the larger rivers, the Colorado, Rio Grande, Pecos, and Gila, are generally associated with winter storms of low intensity, wide extent, and long duration.

The danger of using 24-hr records from nonrecording rain gages should be emphasized. Unless the times of beginning and ending of precipitation are noted carefully at the time of observation, the total rainfall is of little value in any intensity study.

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DISCUSSIONS

EFFECT OF TURBULENCE ON SEDIMENTATION

Discussion

By A. M. Gaudin, Esq.

A. M. Gaudin,[4] Esq.[4a]—A particularly significant attempt to analyze the problem of sedimentation under turbulent conditions is contained in the paper by Mr. Dobbins. The discussion of the distribution of particles in a vessel in which the turbulence varies along a vertical coordinate only is convincing indeed in the agreement of theory and practice. Of course, the broader problem of the distribution of particles in a flowing stream is what the author is looking toward, but has not yet undertaken to solve. It is to be hoped that this will be done in subsequent papers.

Systems in which particles of various sizes, specific gravities, and shapes occur simultaneously have been excluded from analysis; yet they represent the more usual condition. Further work should include such studies.

In concluding these very brief comments, the writer wishes to congratulate Mr. Dobbins for this well-documented and splendid contribution to the knowledge of the movement of solids in fluids.

Corrections for *Transactions:* In February, 1943, *Proceedings*, page 239, Eq. 5*b*, change "$A^{-wy\epsilon^{-1}}$" to "$A\ e^{-wy\epsilon^{-1}}$"; and, on page 255, Fig. 11, change "Equation 47" to "Equation 41."

Note.—This paper by William E. Dobbins, Jun. Am. Soc. C. E., was published in February, 1943, *Proceedings*.

[4] Dept. of Metallurgy, Mass. Inst. Tech., Cambridge, Mass.

[4a] Received by the Secretary March 4, 1943.